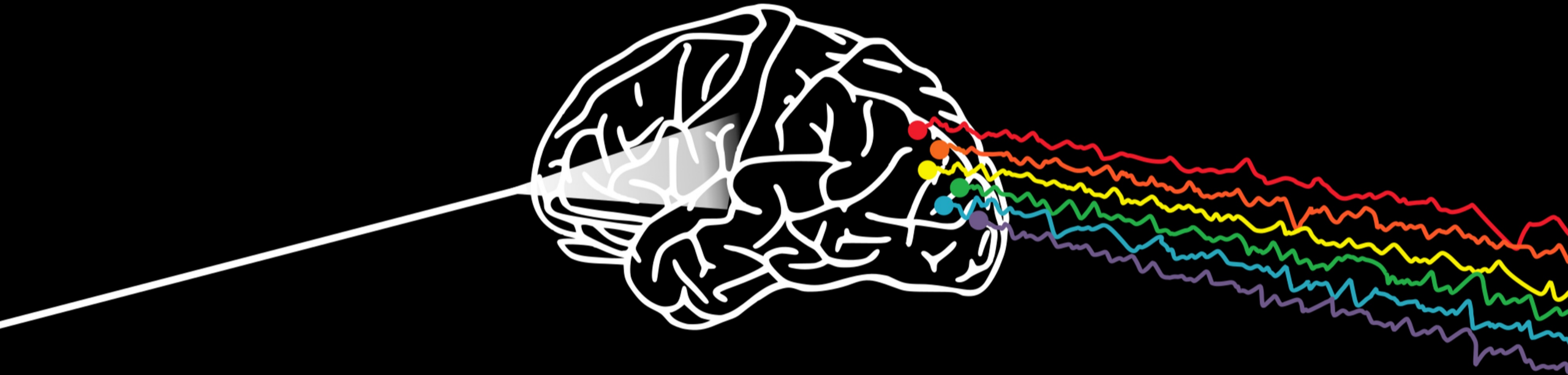


Studying the brain as a complex and nonlinear dynamical system with time-series analysis

CSYS5040

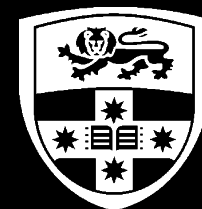
17 September 2024



Annie G. Bryant

PhD Candidate

Dynamics and Neural Systems Lab

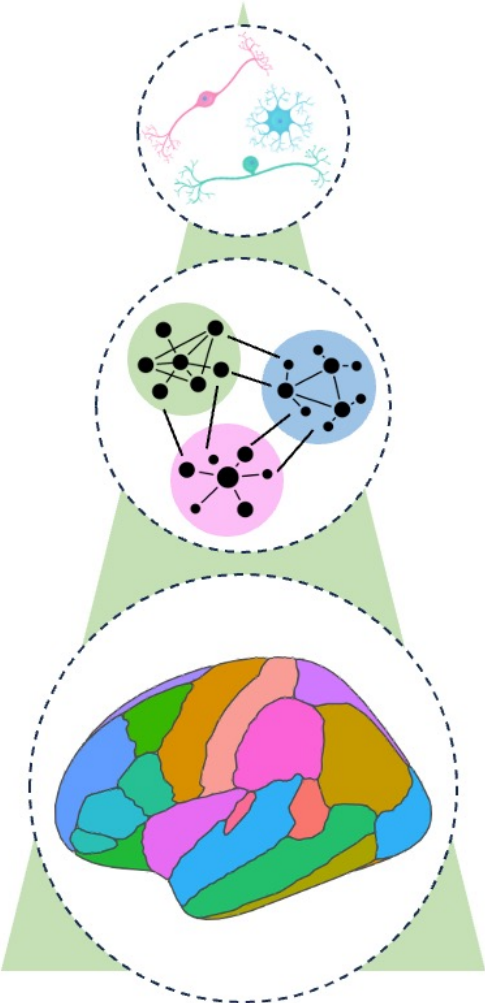


THE UNIVERSITY OF
SYDNEY



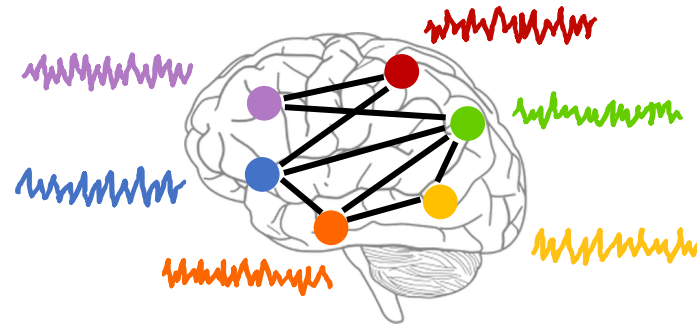
Mechanisms of brain activity across scales

Dynamics emerge at multiple **spatial** and **temporal scales**:



Before we delve into **modelling** or **analysing data** from a given **complex system**, let's think about the **underlying mechanisms** giving rise to **behaviours** from **micro** to **macro scales**.

Door #1:

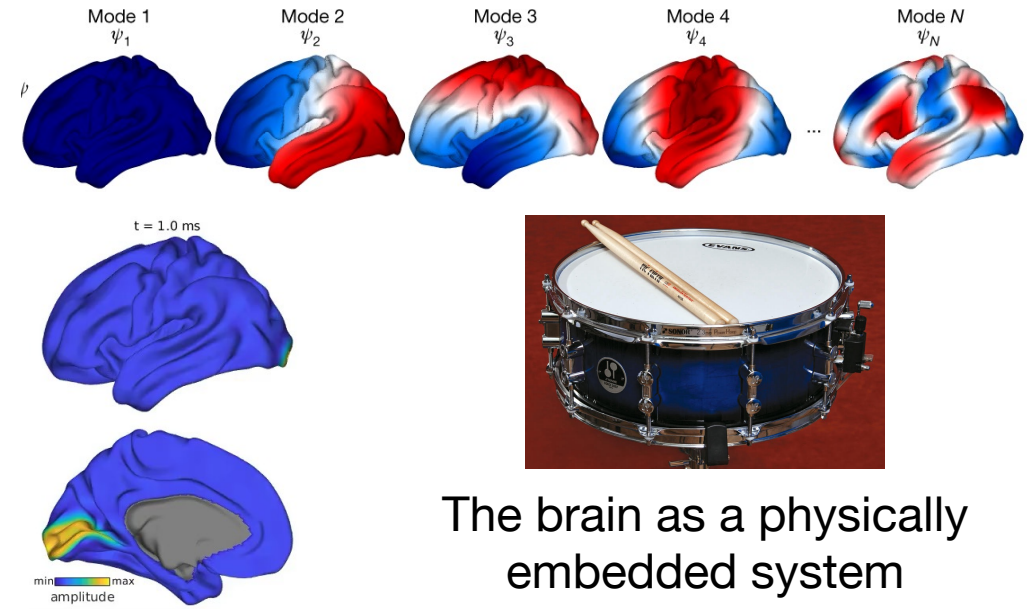


The brain as a network of interacting components



Messages passing across a discrete network

Door #2:

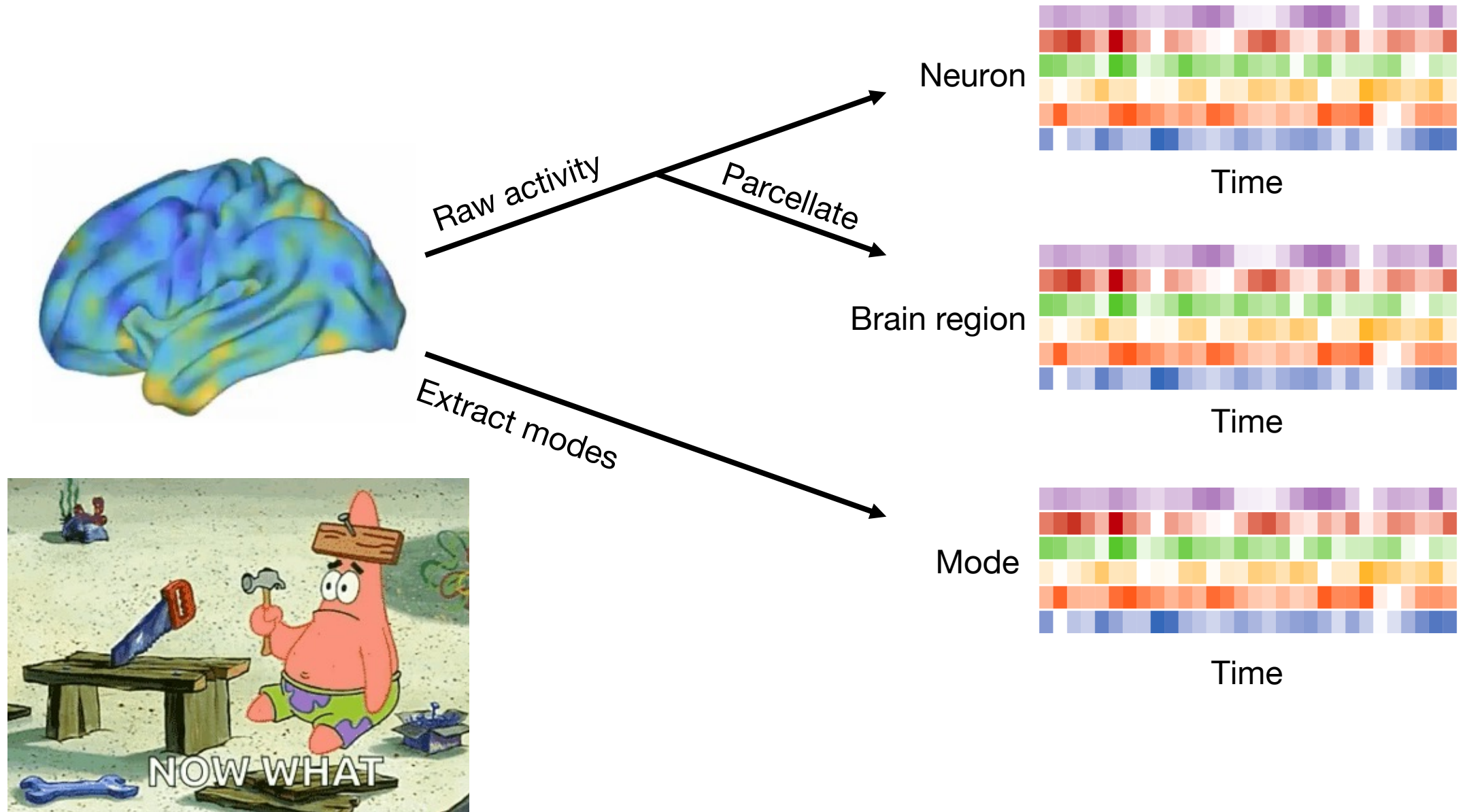


The brain as a physically embedded system

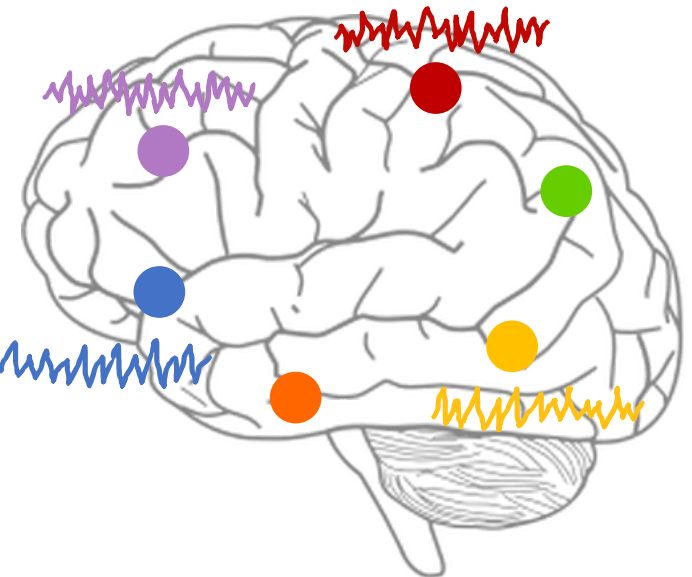


Isotropic propagation across a continuous medium

Different ways of representing brain dynamics



Not just the brain: time-varying complex systems are everywhere!

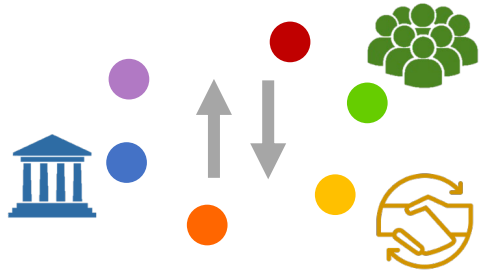


City structure



City properties: density, traffic patterns, crime rates, culture

Economics



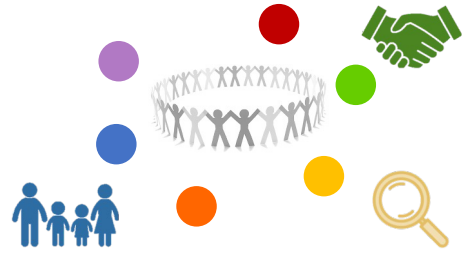
National economy: economic growth, recession

Physics



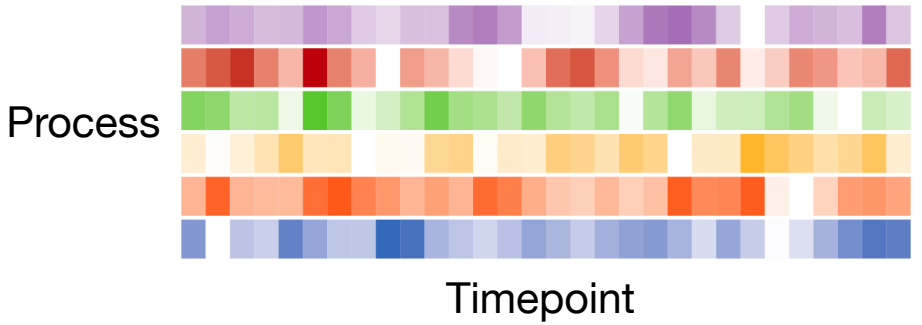
Fluid dynamics: vortices, turbulence

Social networks

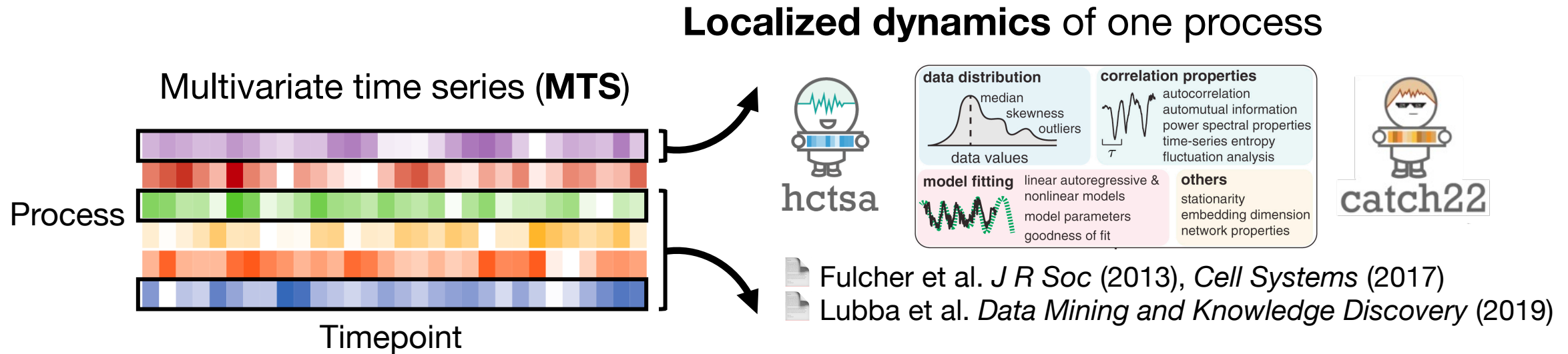


Facebook friends: community formation

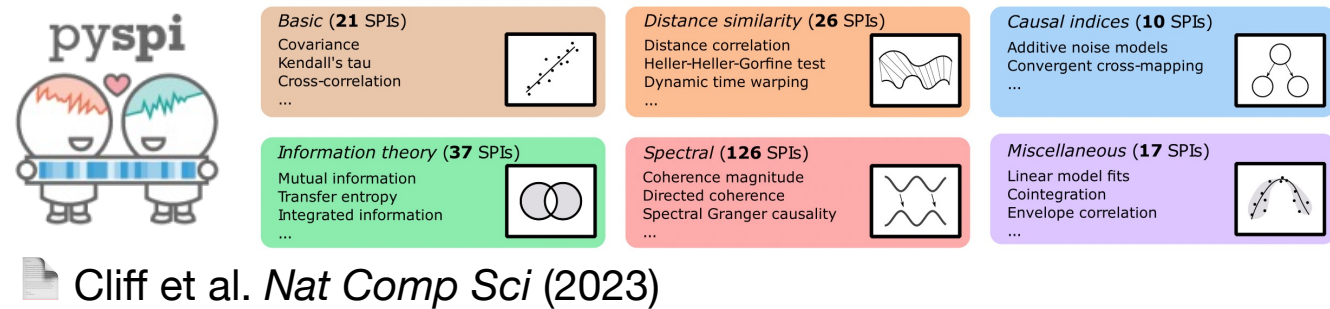
Multivariate time series (MTS)



What does this **generalized representation** offer us?

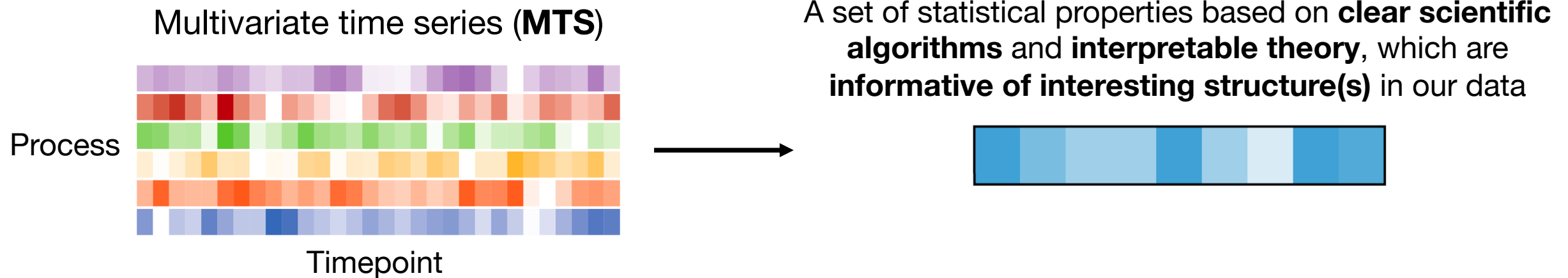


Statistical dependencies between **pairs** of processes



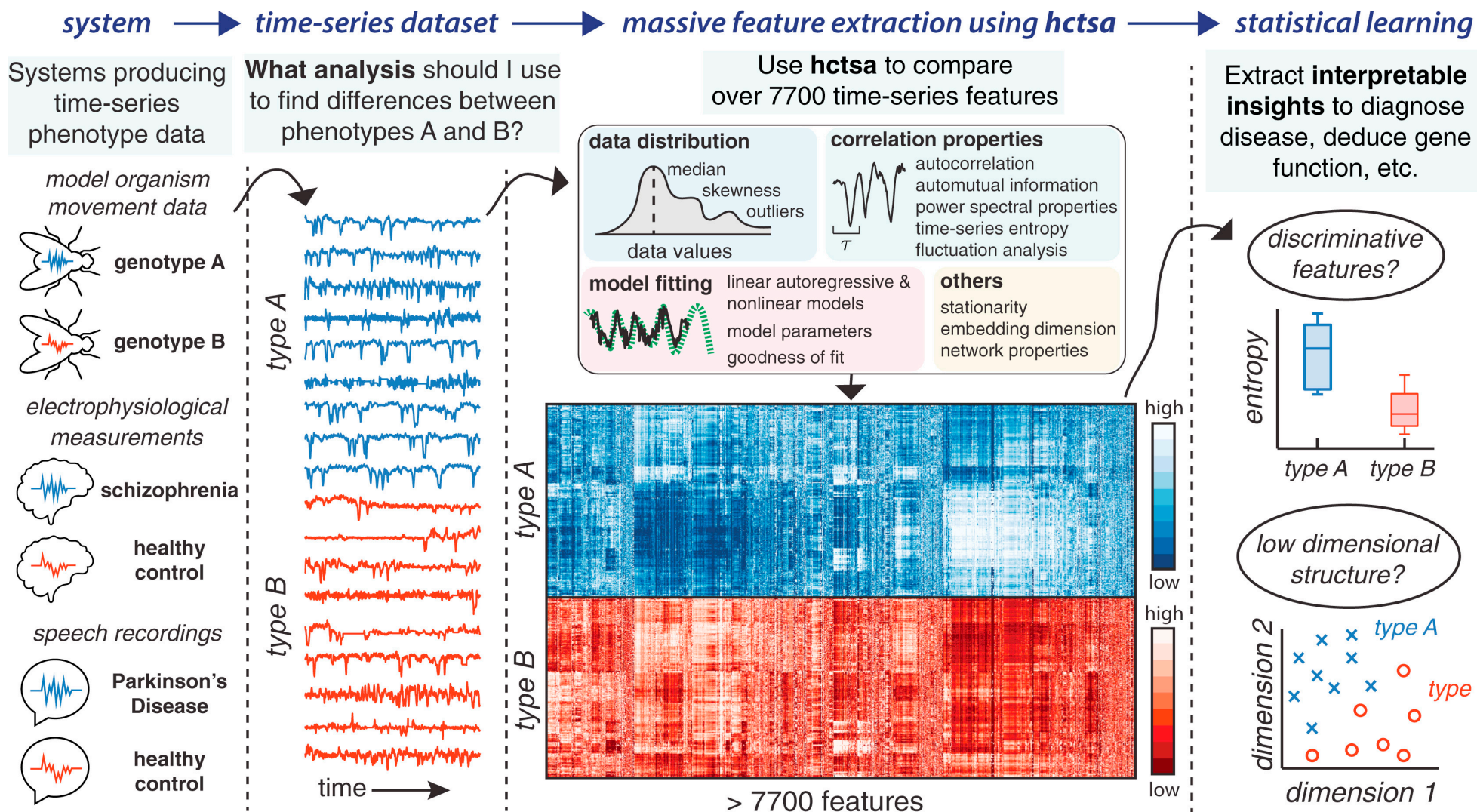
This boils down to a **common goal** 🌡️

Quantifying the desired structure in a multivariate time series:

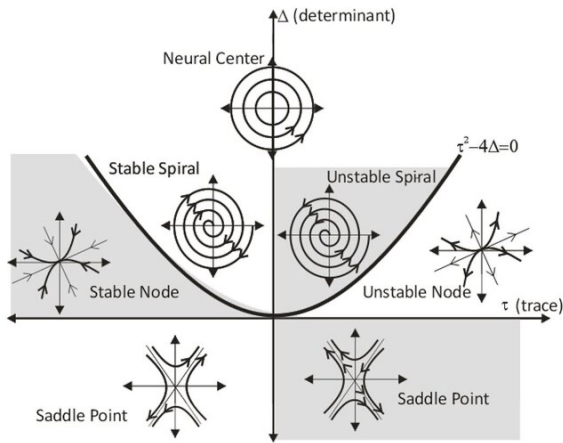


The **highly comparative time-series feature analysis** approach compares across a large library of scientific algorithms (spanning a large and interdisciplinary theoretical literature 📖📖📖📖)

This boils down to a common goal



hctsa and *pyspi* include both **linear** and **nonlinear** features



The arsenal of **linear time series analysis features** begin by **assuming a system with linear structure**, and we think about what such a linear system would do

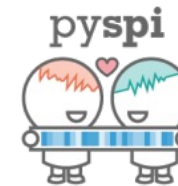
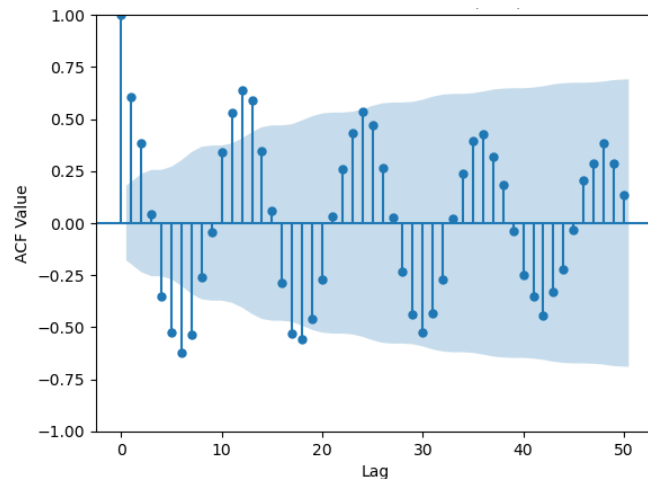


Local dynamics

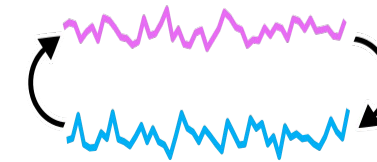


ACF at different lags, local forecast based on rolling average, AR models, basic distributional properties (mean, variance)

A linear system is **fully** captured by its **autocorrelation function (ACF)**

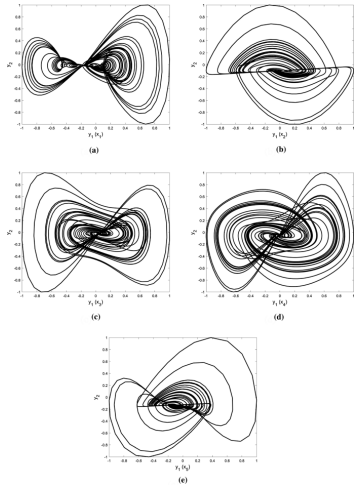


Pairwise coupling

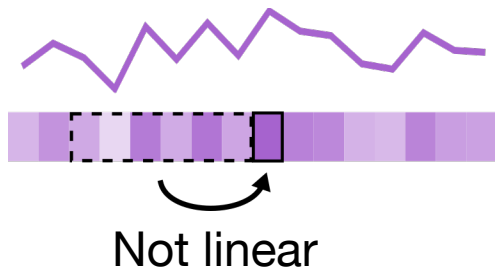


Pearson correlation, Granger causality, Euclidean distance, linear model fits, power envelope correlation

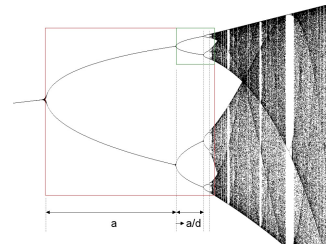
hctsa and *pyspi* include both **linear** and **nonlinear** features



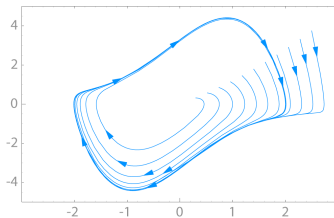
Nonlinear time series analysis features do not make assumptions about the structure of the system, which can be additionally summarized by e.g., polynomial functions



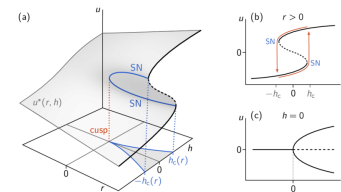
Small changes to the input can give rise to:



Chaos



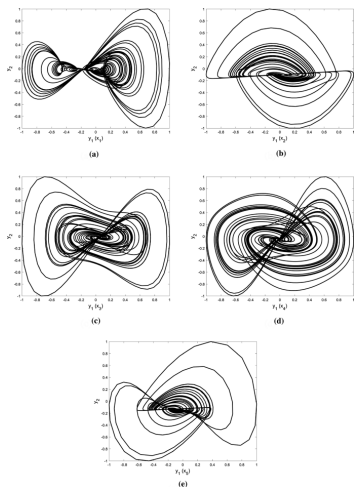
Oscillations



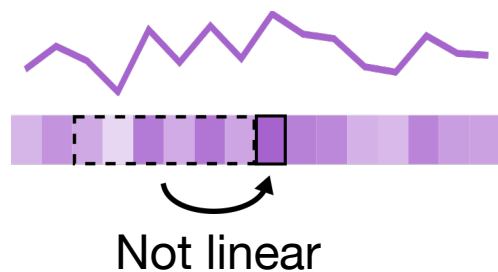
Bifurcations



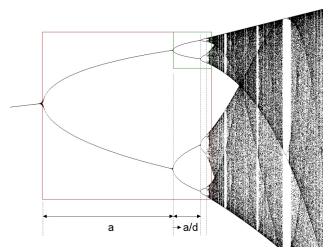
hctsa and *pyspi* include both **linear** and **nonlinear** features



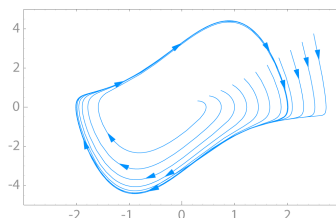
Nonlinear time series analysis features do not make assumptions about the structure of the system, which can be additionally summarized by e.g., polynomial functions



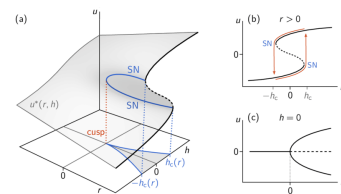
Small changes to the input can give rise to:



Chaos



Oscillations



Bifurcations

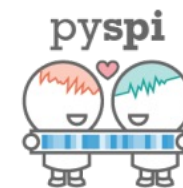


hctsa

Local dynamics

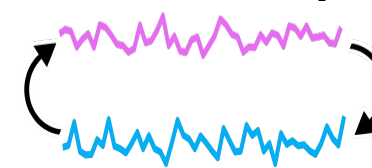


Automutual information, Lyapunov exponent, fractional dimensionality, phase-space entropies, embedding distance



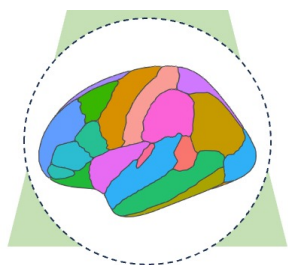
pyspi

Pairwise coupling

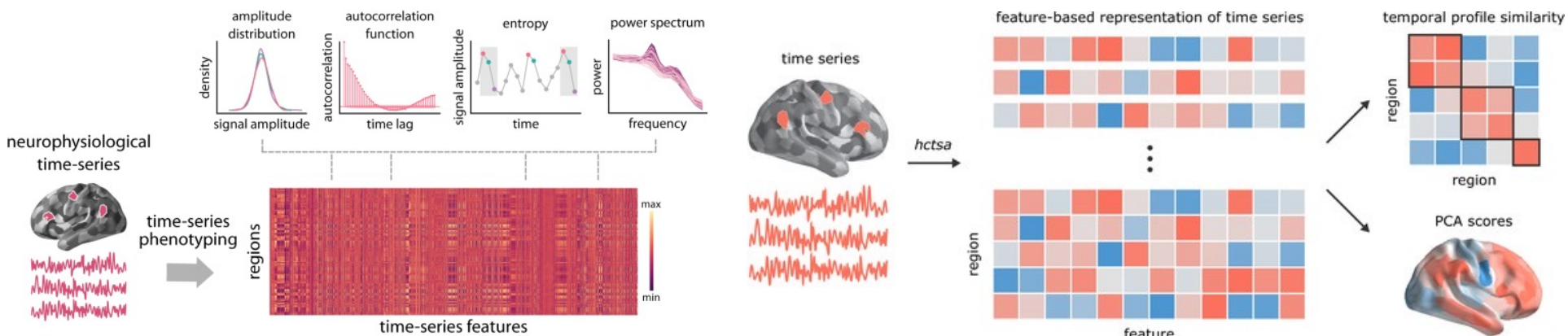


Transfer entropy, dynamic time warping, additive noise modelling, phase lag index

Nonlinearity in brain dynamics (?)



Real-world systems that we study, like the **brain**, extend **beyond** the behaviors that a linear system can exhibit; we therefore define them as **nonlinear**.

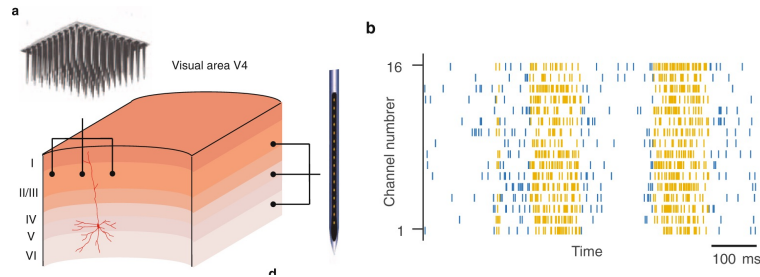
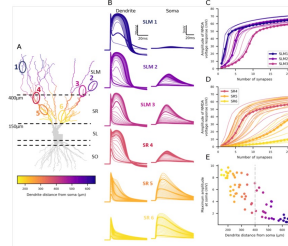
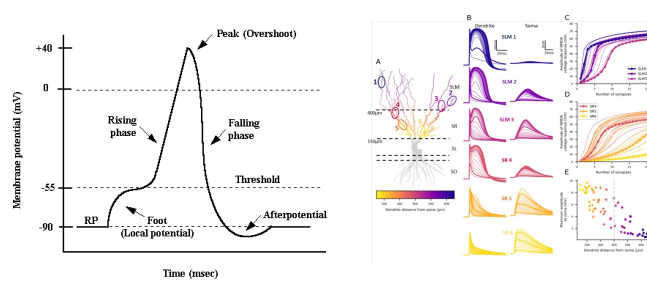


Applications of *hctsa* to **MEG (left)** and **fMRI (right)** data have shown that **linear properties** of local dynamics tend to dominate the **principal brain-wide axes of temporal variation** of the brain at rest.



When your brain is at rest – as in not actively performing a cognitive task – it is sitting close to an equilibrium, where the **governing dynamics are approximately linear**. To detect nonlinear dynamics with **functional neuroimaging**, do we need to increase the **temporal precision** and/or **perturb** the brain away from resting equilibrium?

Do we still see nonlinearity at the macroscale?



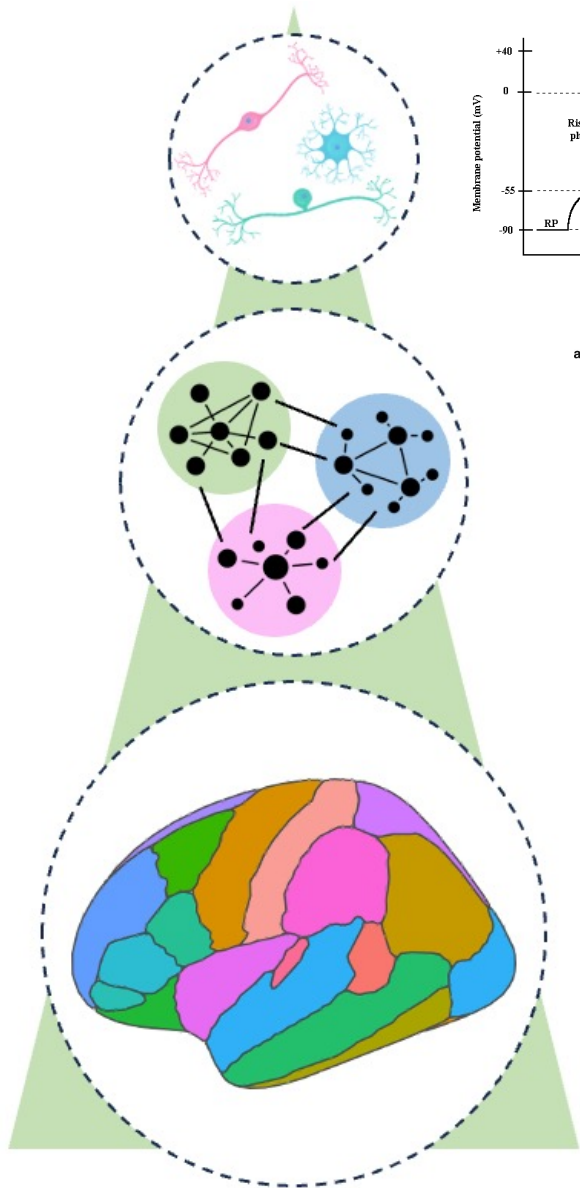
Article | [Open access](#) | Published: 11 December 2023

Macroscopic resting-state brain dynamics are best described by linear models

[Erfan Nozari](#), [Maxwell A. Bertolo](#), [Jennifer Stiso](#), [Lorenzo Caciagli](#), [Eli J. Cornblath](#), [Xiaosong He](#), [Arun S. Mahadevan](#), [George J. Pappas](#) & [Dani S. Bassett](#)

Nature Biomedical Engineering **8**, 68–84 (2024) | [Cite this article](#)

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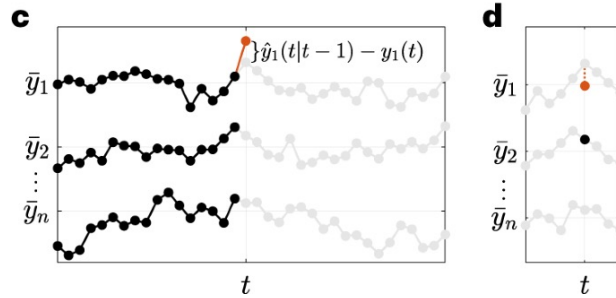
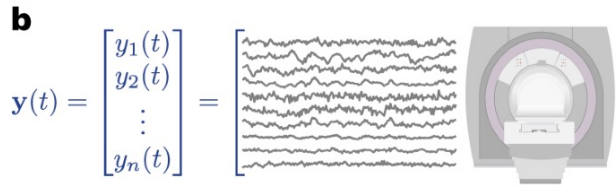
<https://www-archiv.fdm.uni-hamburg.de/online/library/crone/3028/membrane/mempot.html>;

Humphries et al. *Neuroscience* (2023); Shi et al. *Nat Comms* (2022)

Do we still see nonlinearity at the macroscale?

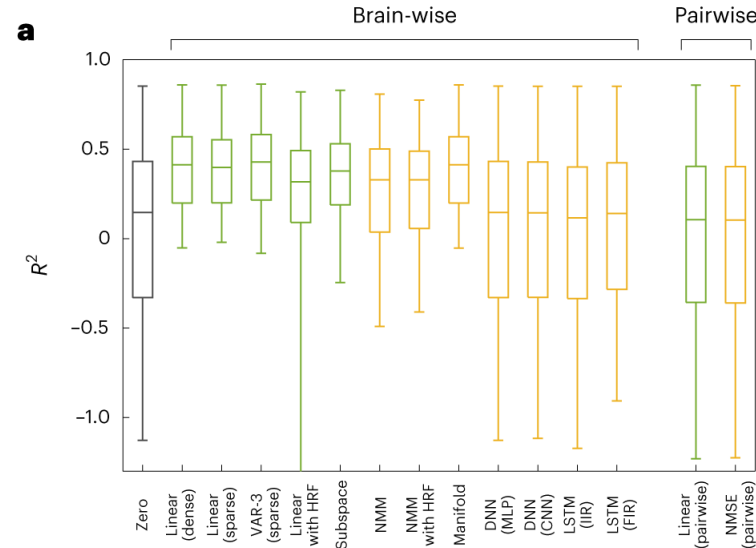
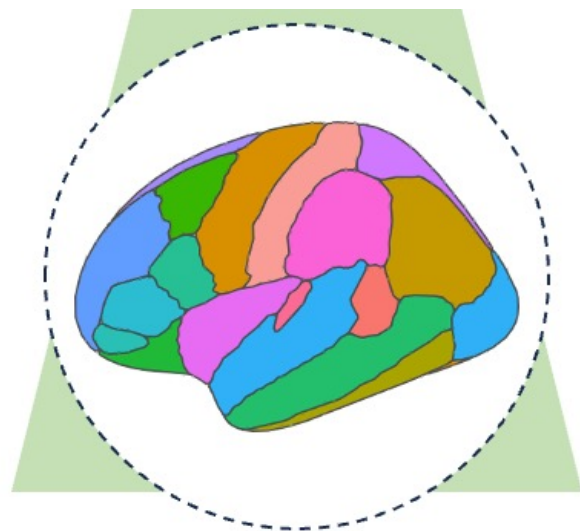
a

$$\mathcal{M} : \begin{cases} \dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + \mathbf{e}_1(t), & \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y}(t) = h(\mathbf{x}(t)) + \mathbf{e}_2(t) \end{cases}$$



With an AR(1) process:
 $x_{t+1} = f(x_t)$ with a linear function,
 fully specified by the
autocorrelation function at lag 1

AC(1) is a “**sufficient statistic**”
 here – the summary of past
 observations that contains info
 needed to predict the future
 values of x



Article | [Open access](#) | Published: 11 December 2023

Macroscopic resting-state brain dynamics are best described by linear models

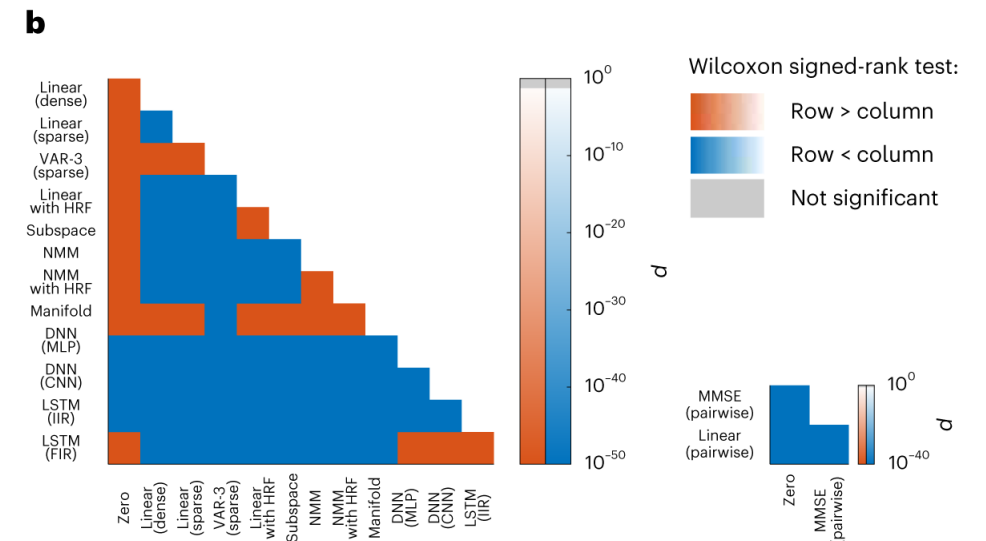
[Erfan Nozari](#), [Maxwell A. Bertolero](#), [Jennifer Stiso](#), [Lorenzo Caciagli](#), [Eli J. Cornblath](#), [Xiaosong He](#), [Arun S. Mahadevan](#), [George J. Pappas](#) & [Dani S. Bassett](#) ✉

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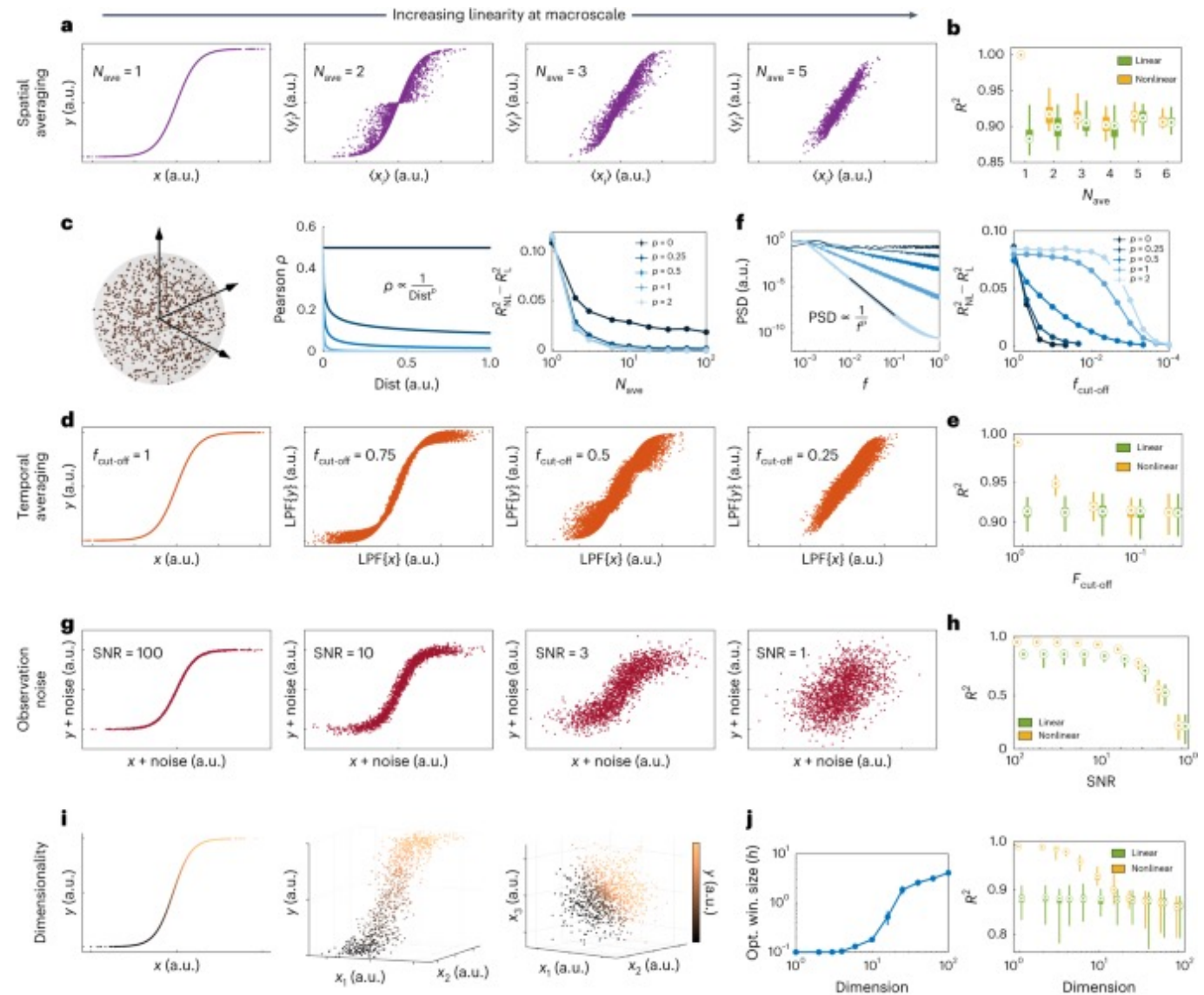
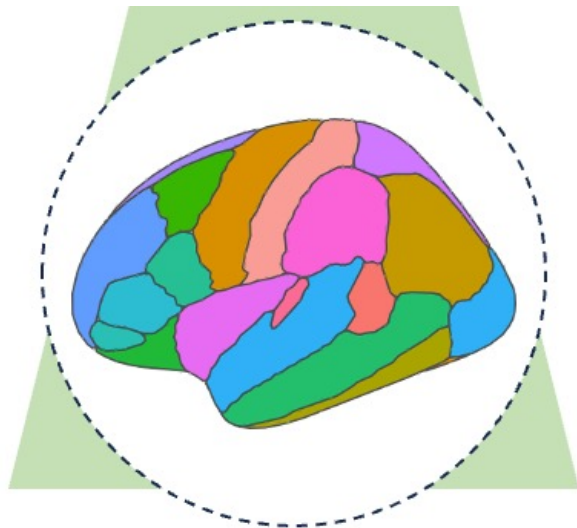
Linear autoregressive (AR) models



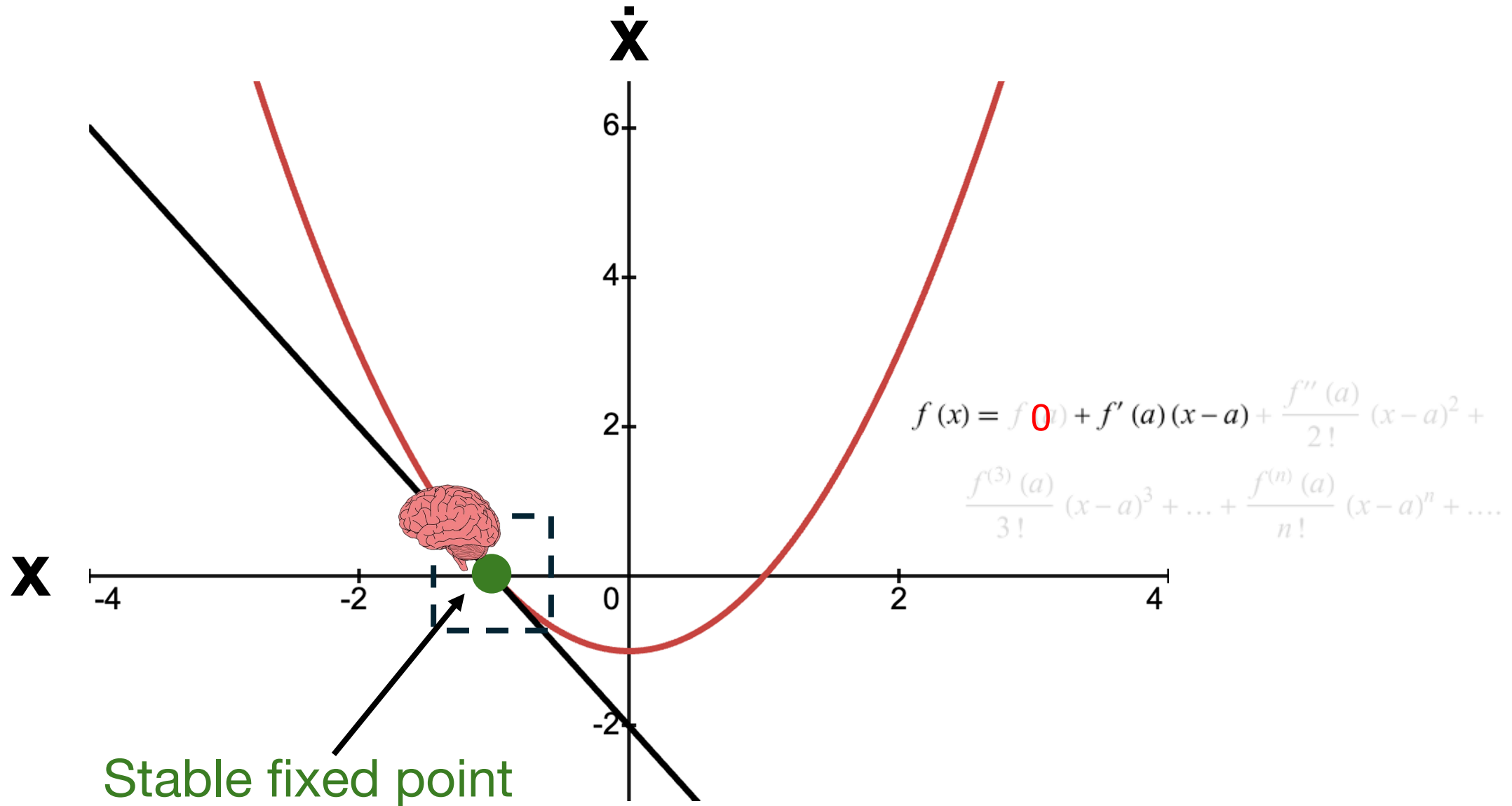
Do we still see nonlinearity at the macroscale?

4 “linearizing effects” that we can observe with functional neuroimaging at the whole-brain scale:

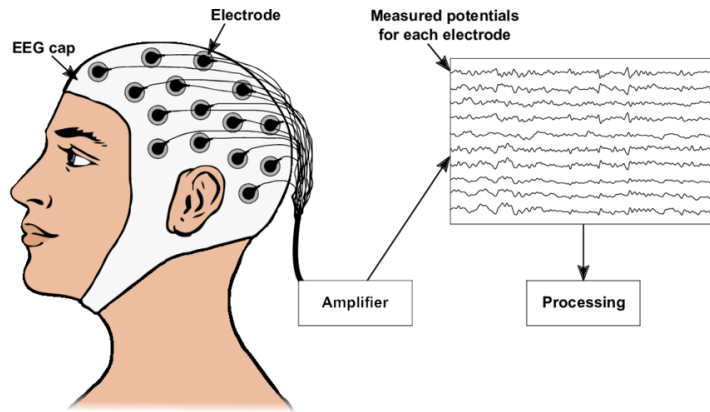
1. Spatial averaging
2. Temporal averaging
3. Observational noise
4. Dimensionality



Nonlinear systems can be locally linear

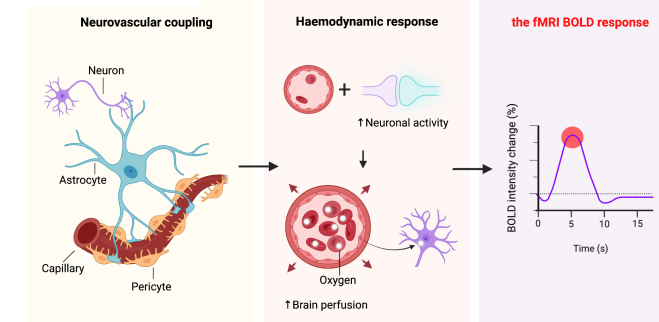
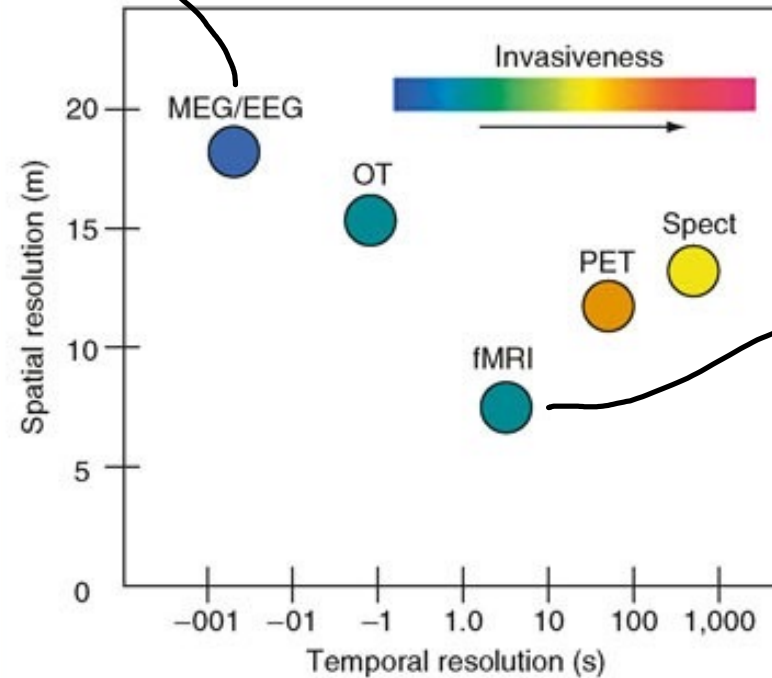


Brain imaging modalities exhibit a **spatial/temporal resolution tradeoff**



Electroencephalography (**EEG**) **directly** measures **activity of neural masses** (populations of neurons) at **high temporal precision**, but spatial resolution is **limited to the cortical surface**

↑ Temporal ↓ Spatial

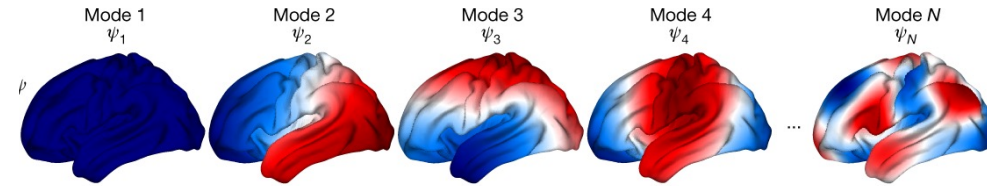


Functional magnetic resonance imaging (**fMRI**) **indirectly** measures neural activity from the **cortex down to deep gray matter nuclei** via **hemodynamic coupling**

↓ Temporal ↑ Spatial

Time-series analysis with realistic brain activity datasets

>7,000 features



Tens to hundreds of modes or parcels

Glasser (360 parcels)	Gordon (333 parcels)	NeuroSynth (113 parcels)	Power (280 parcels)	Schaefer (100, 200 & 400 parcels)	Yeo (51 parcels)
On the Surface (Glasser et al., 2016)	On the Surface (Gordon et al., 2016)	In the Volume www.neurosynth.org	In the Volume (Power et al., 2011)	On the Surface (Schaefer et al., 2018)	On the Surface (Yeo et al., 2011)
<ul style="list-style-type: none"> 1 "Default" 2 "Dorsal Attention" 3 "FrontoParietal" 4 "CinguloOpercular" 	<ul style="list-style-type: none"> 1 "Default Mode" 2 "Dorsal Attention" 3 "Fronto-Parietal" 4 "CinguloOpercular" 	<p>KEYWORDS:</p> <ul style="list-style-type: none"> "Default Mode" "Dorsal Attention" "Fronto-Parietal Task Control" "Control network" "Salience Network" 	<ul style="list-style-type: none"> 1 "Default Mode" 2 "Dorsal Attention" 3 "Fronto-Parietal Task Control" 4 "CinguloOpercular" 	<ul style="list-style-type: none"> 1 "Default" 2 "Dorsal Attention" 3 "FrontoParietal Control" 4 "Ventral Attention" 	<ul style="list-style-type: none"> 1 "Default" 2 "Dorsal Attention" 3 "FrontoParietal Control" 4 "Ventral Attention"



Sample sizes of a few hundred at the most in neuroimaging studies in general

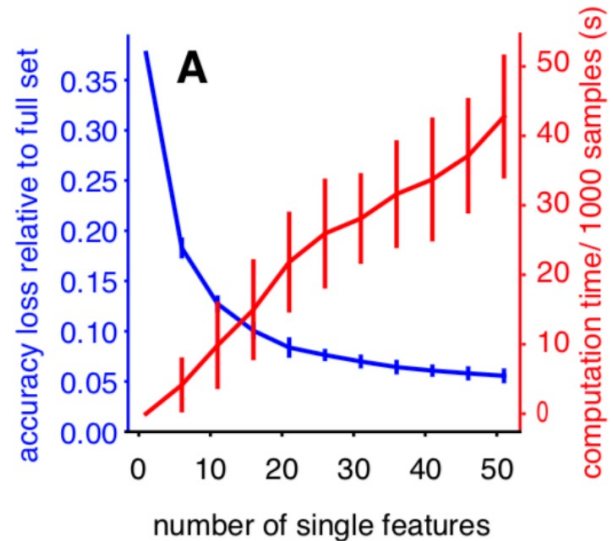


Time-series analysis with realistic brain activity datasets

catch22

Evaluate features across **93 diverse time-series classification tasks**

We can usually get away with using **just 22 features**.

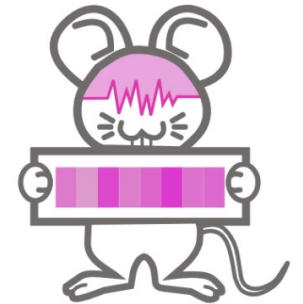
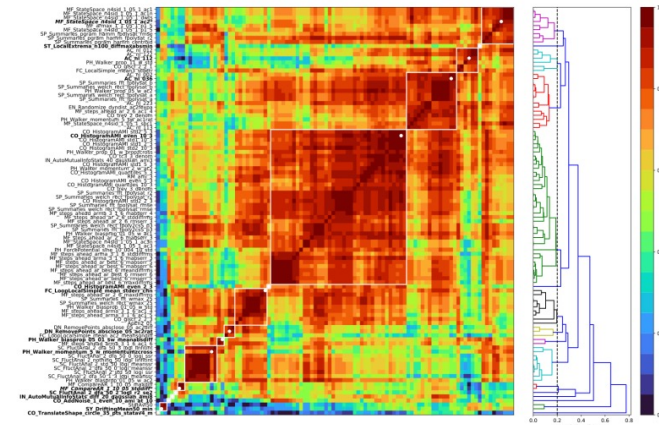


Lubba et al. *catch22: CAnonical Time-series CHaracteristics*. *Data Min Knowl Disc* (2019).

catchaMouse16

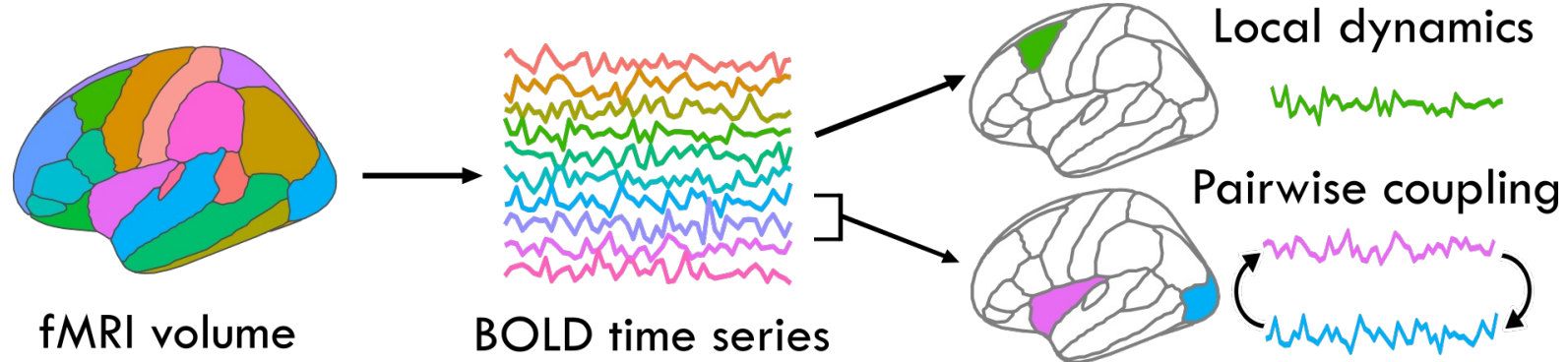
Feature evaluation across **12 mouse fMRI classification tasks**

We can usually get away with using **just 16 features**.

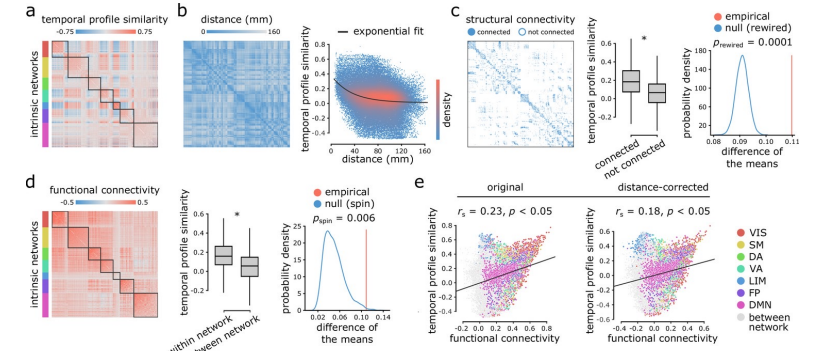


Alam et al. Canonical time-series features for characterizing biologically informative dynamical patterns in fMRI. *bioRxiv* (2024).

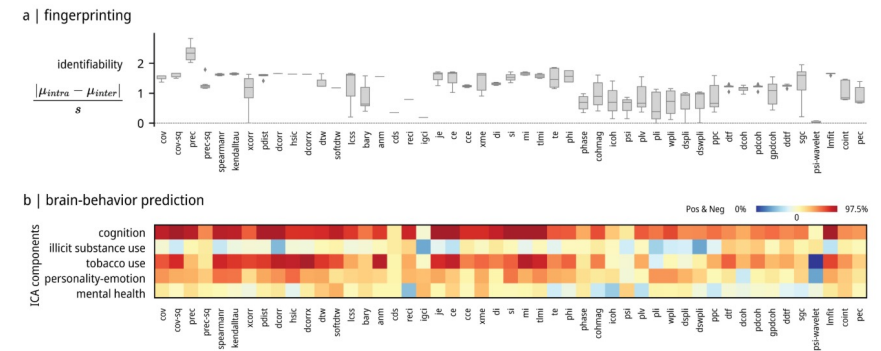
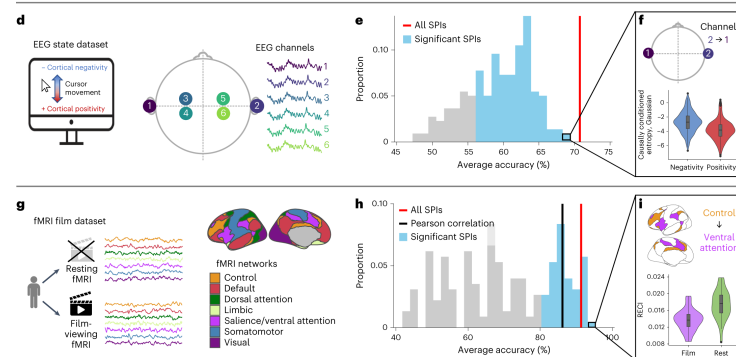
A shameless plug case study on time-series analysis for fMRI



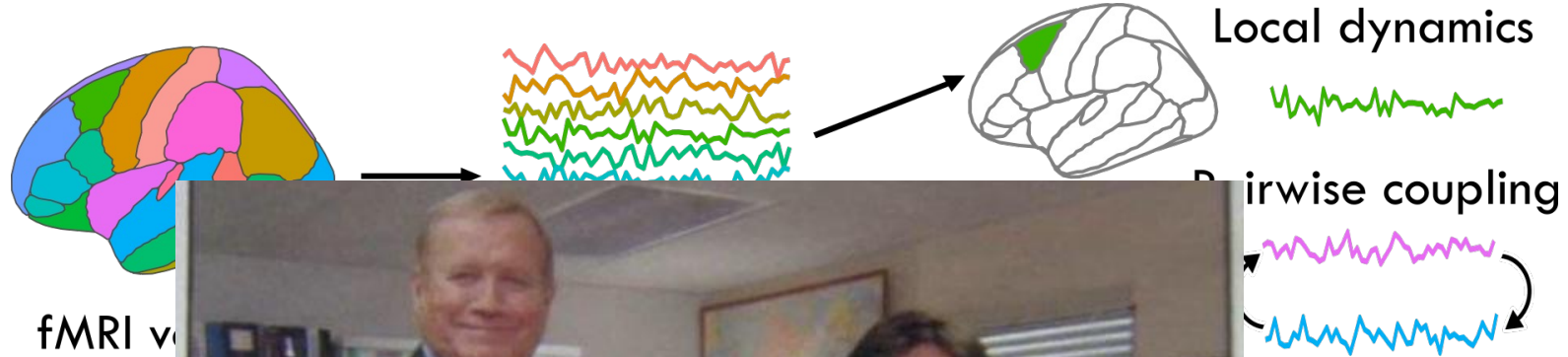
Shafiei et al. *eLife* 2020 (pictured),
Nat Comms 2023



Cliff et al. *Nat Comp Sci* 2023 (left)
 Liu et al. *bioRxiv* 2024 (right)



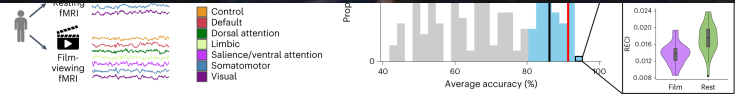
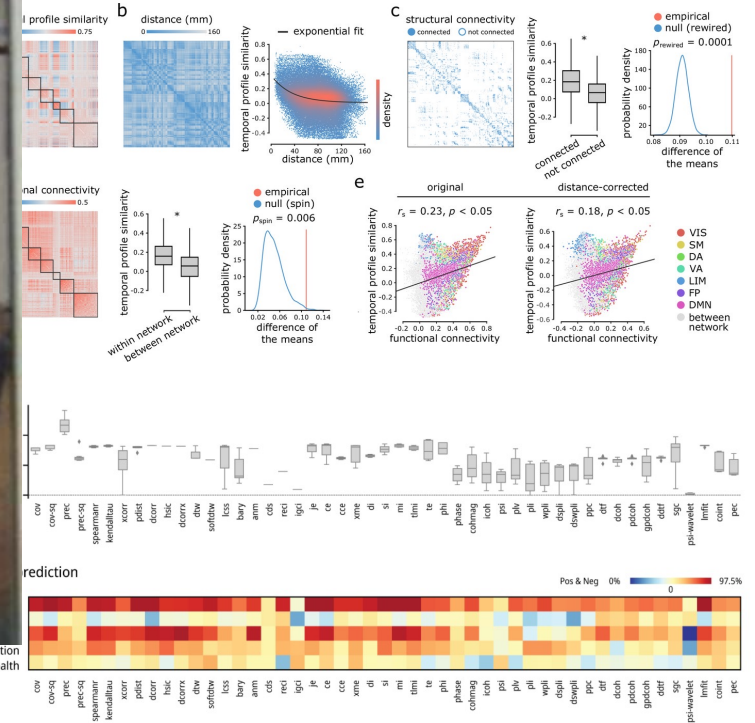
A shameless plug case study on time-series analysis for fMRI



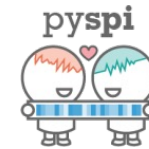
Shafiei et al. *eLife* 2020 (picture)
 Nat Comms 2023



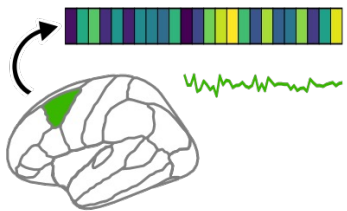
Cliff et al. *Nat Comp Sci* 2023
 Liu et al. *bioRxiv* 2024 (right)



Classifying neuropsychiatric disorder cases vs. healthy controls

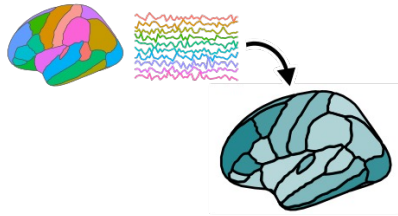


i. Local dynamics in an individual region



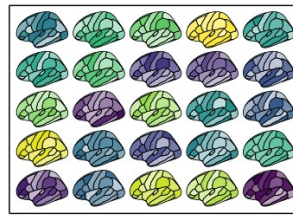
(A_{region})

ii. Whole-brain maps of an individual feature



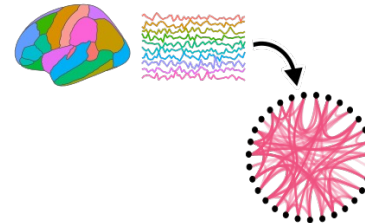
(A_{feature})

iii. Whole-brain maps of all features



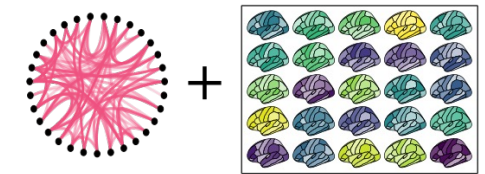
($A_{\text{uni_combo}}$)

iv. FC across all region pairs with one SPI

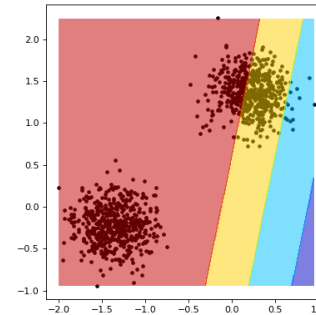
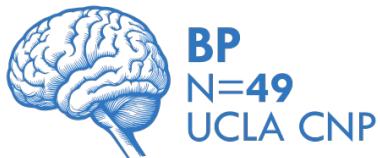


(A_{FC})

v. FC across all region pairs by SPI plus all whole-brain maps of local dynamics



($A_{\text{FC_combo}}$)



Source: scikit-learn

Linear support vector machine (SVM)

- Balanced accuracy
- Inverse probability weighting

× 10-fold CV
× 10 repeats

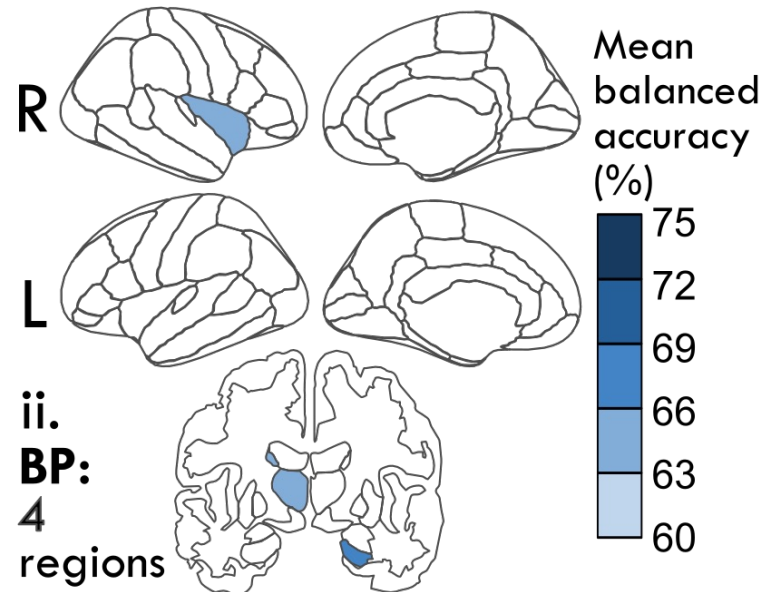
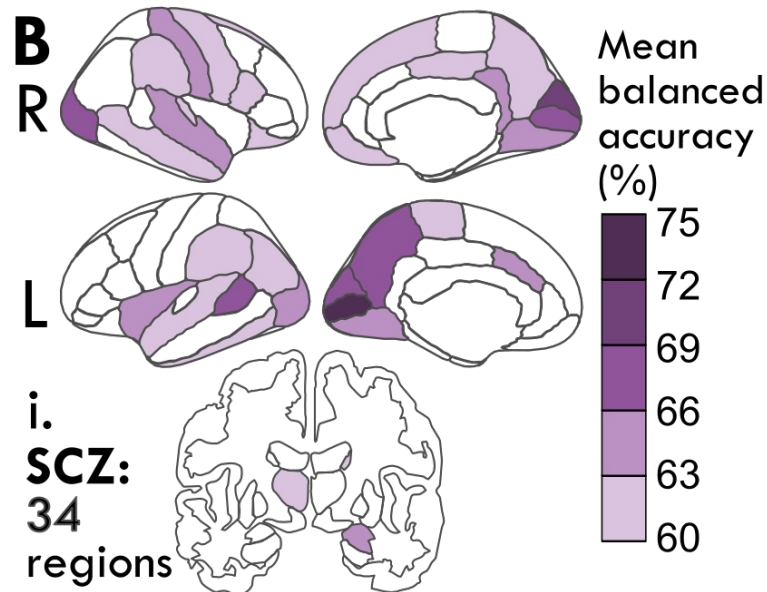
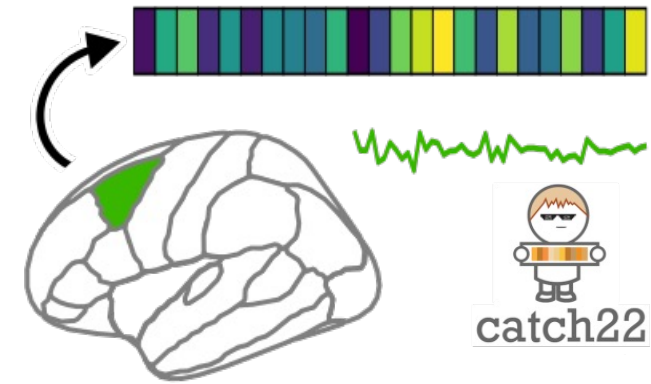



Data preprocessed by Traut et al. *NeuroImage* (2022)


Data preprocessed by Dr Kevin Aquino


Interpretable spatial maps of region-specific dysfunction

Dynamical signatures of resting-state activity in **individual brain regions** can distinguish patients from controls in schizophrenia and bipolar disorder



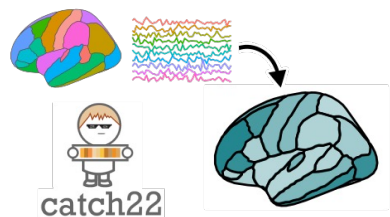
Gene expression 

Anatomical changes 

Stimulation analysis 

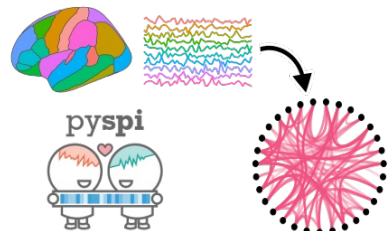
Highlighting **linear features** for resting-state fMRI analysis

ii. Whole-brain maps of an individual feature

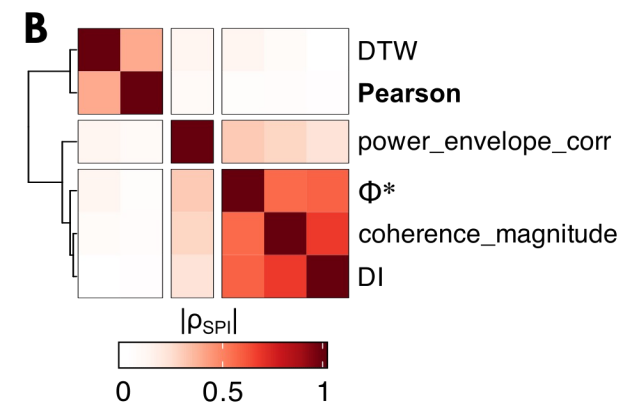
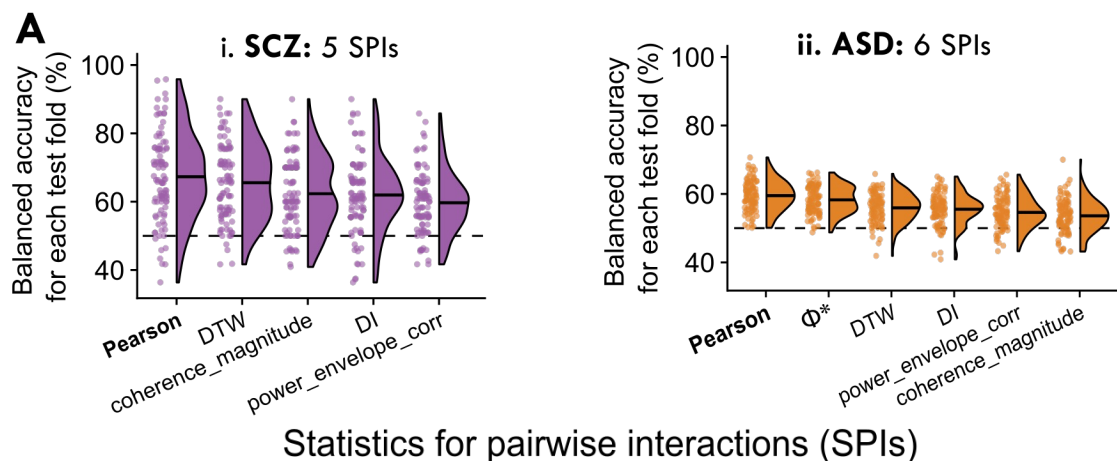
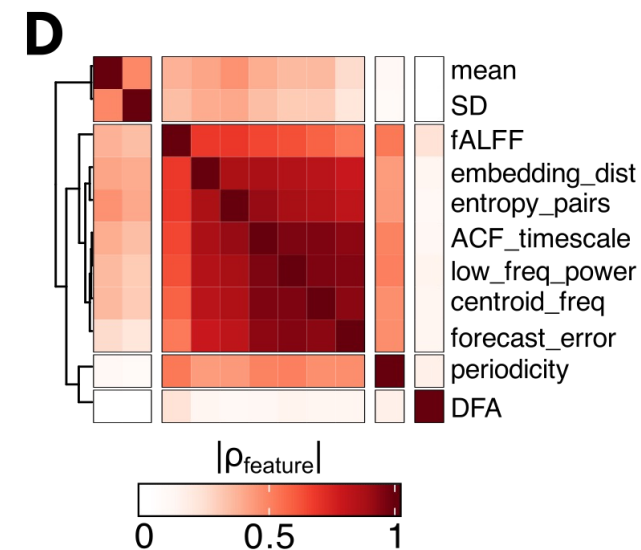
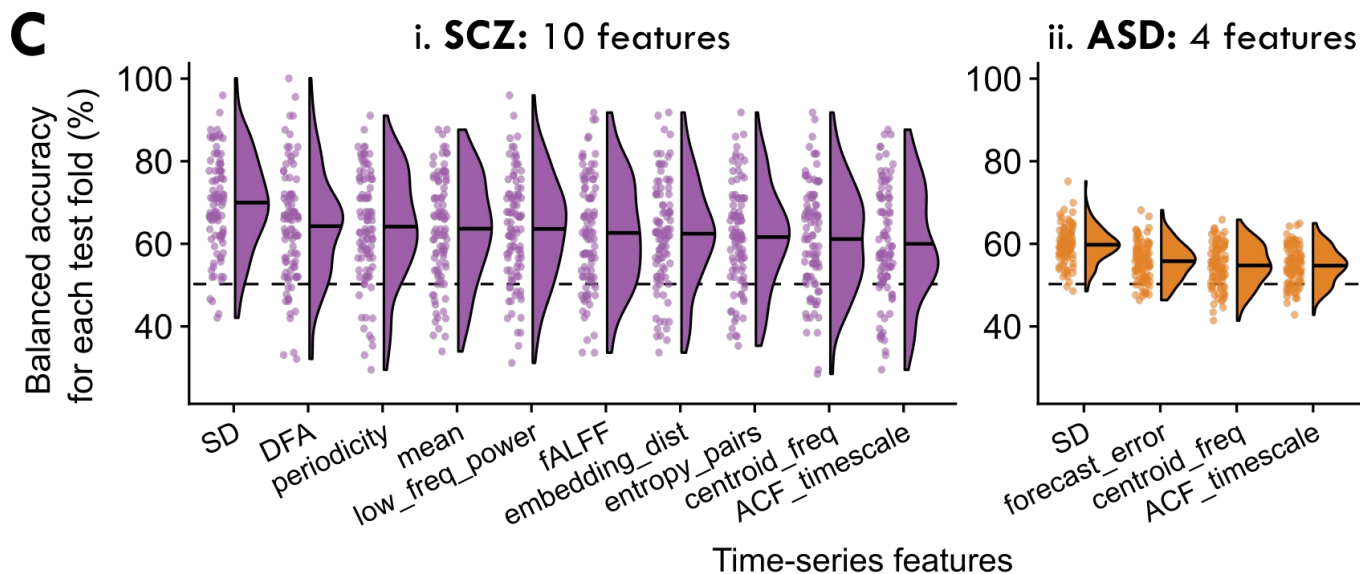


(A_{feature})

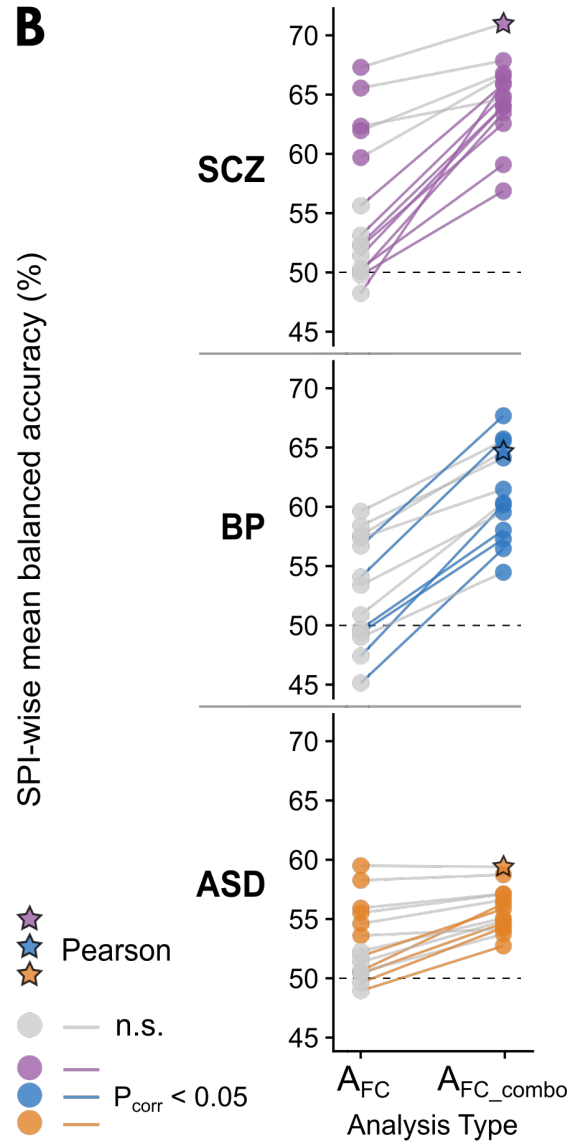
iv. FC across all region pairs with one SPI



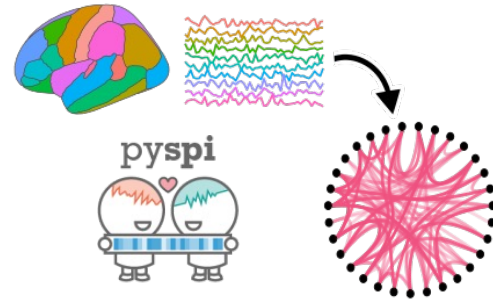
(A_{FC})



The benefit of **integrating** local dynamics **and** pairwise coupling

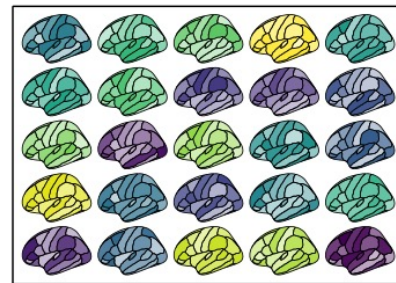


iv. FC across all region pairs with one SPI



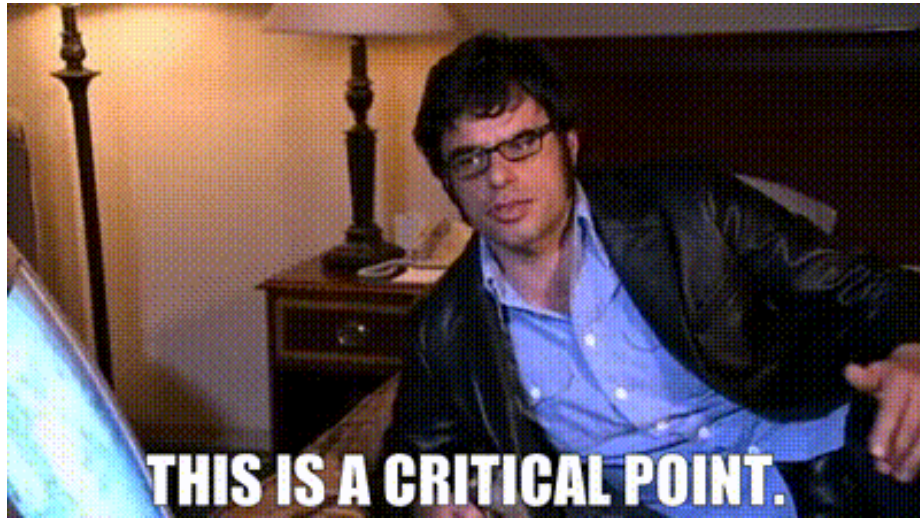
+


iii. Whole-brain maps of all features

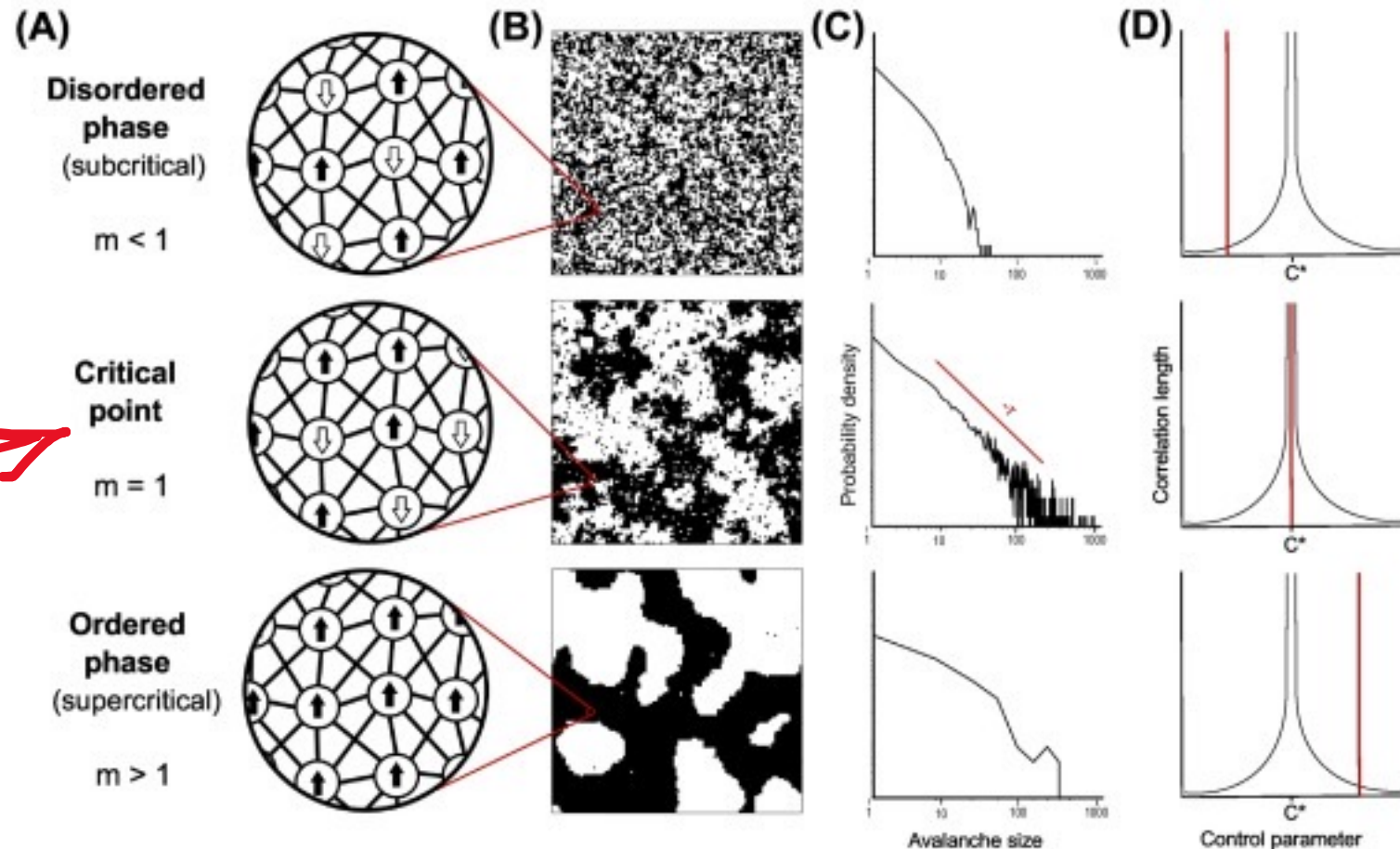


Most **functional connectivity** metrics are more informative with the inclusion of **brain-wide maps of local regional dynamics**

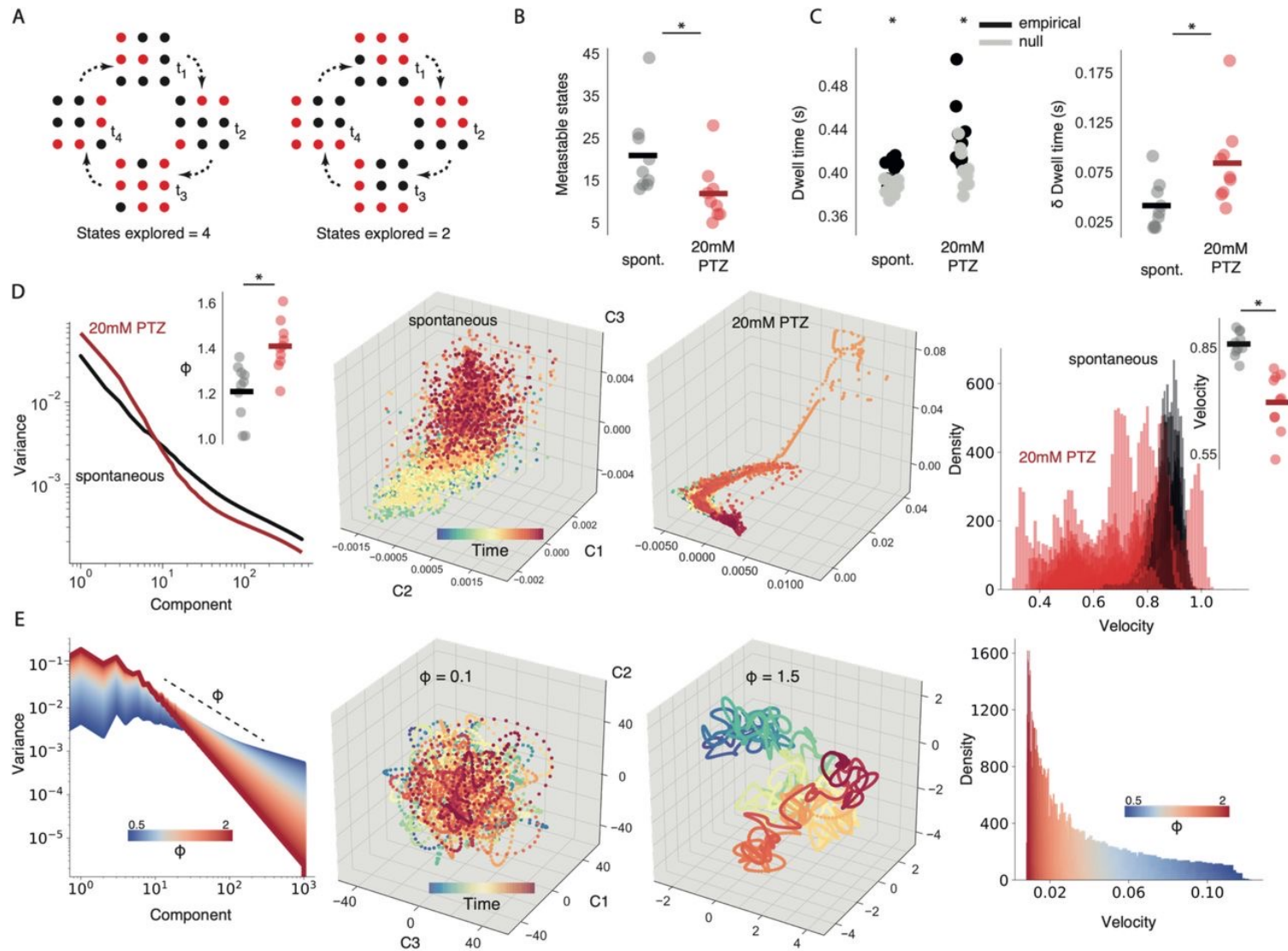
Segue to: Tracking the **distance to criticality**



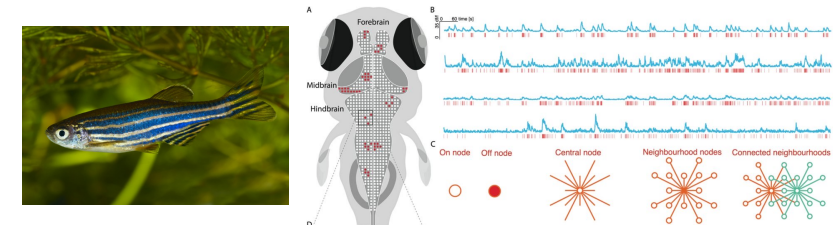
Ising model: nodes of the lattice can be thought of as individual neurons/neuronal ensembles 



We see patterns of **criticality** in **brain dynamics**

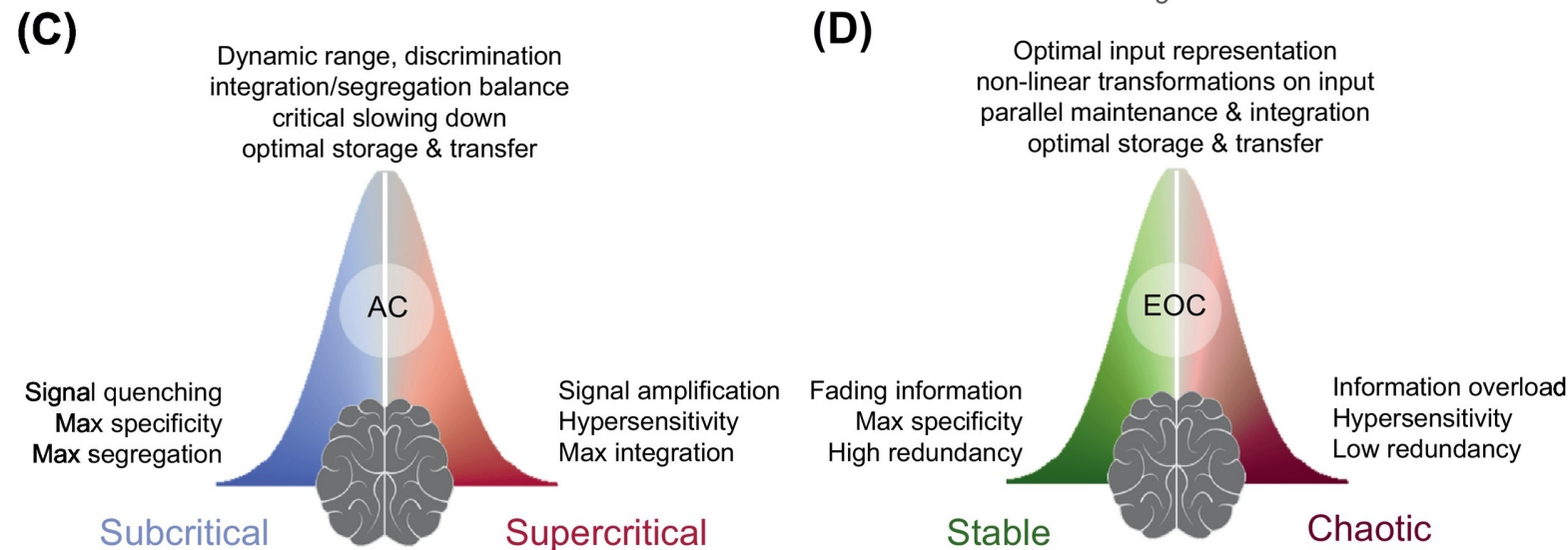
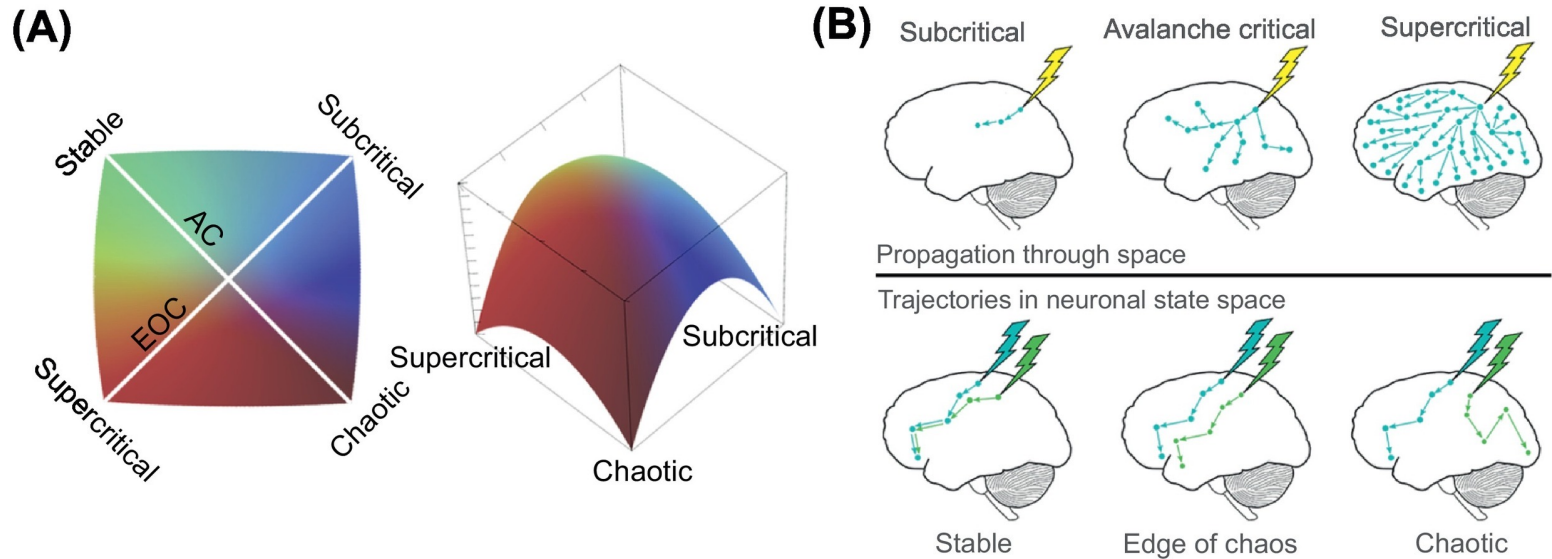


"(A) A **critical system** can explore a **greater subset of its possible brain states** ... while a non-critical system will explore a more limited subset."



Zebrafish models are used to study how **epilepsy** induces changes to **excitation/inhibition balances** that push the brain away from **criticality**.

We see patterns of **criticality** in **brain dynamics**

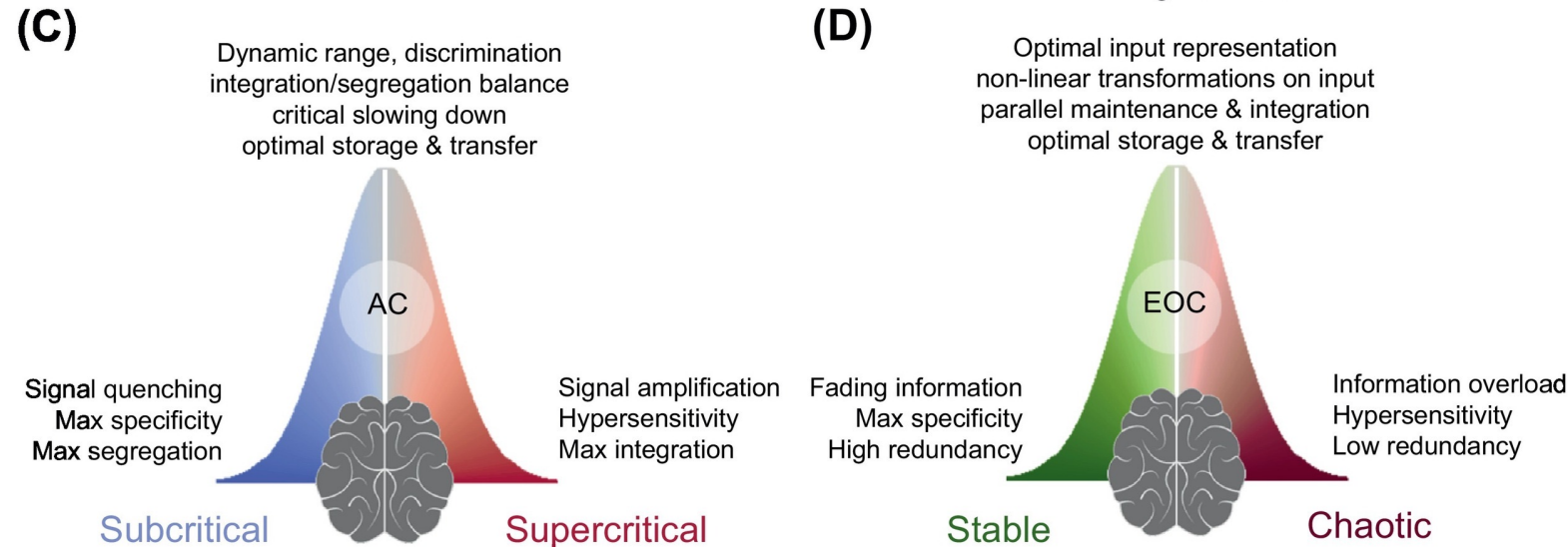
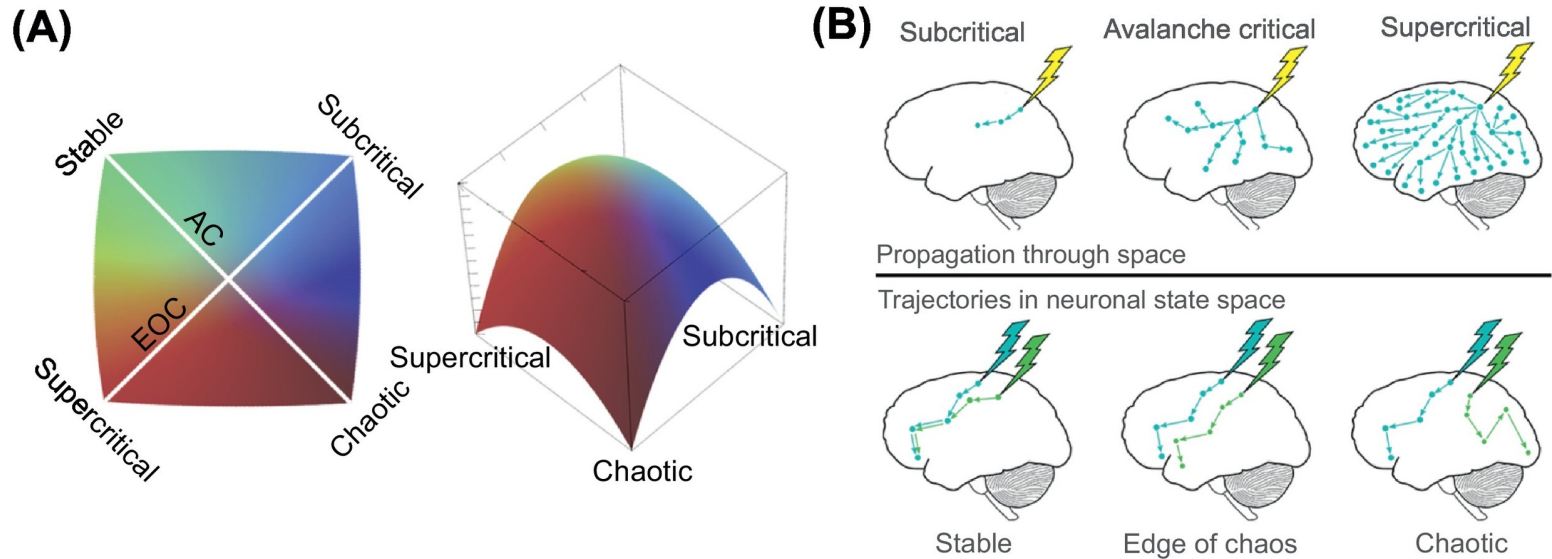


Trends in Neurosciences

“The **distance to criticality** presents a **promising and underexploited** biological parameter for characterizing **cognitive differences** and **mental illness.**”

“...these ideas underscore the relevance of the **distance to criticality** for **cognition**: this distance may be **dynamically varied** at a moment-to-moment timescale in order to **flexibly adapt** to task requirements.”

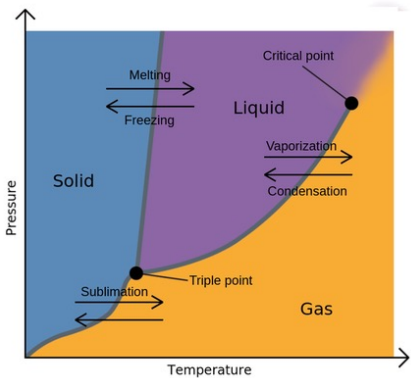
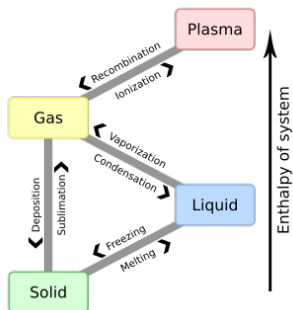
We see patterns of **criticality** in **brain dynamics**



“[T]he distance to criticality may constitute a well-placed biological variable for understanding both **healthy** and **pathological changes** in cognition and behavior, one that forms a **direct mechanistic link** between **neurons** and **neuronal ensembles** at the **micro scale** and **computation** at the **macro scale**.”

Distance to criticality is relevant **beyond the brain**

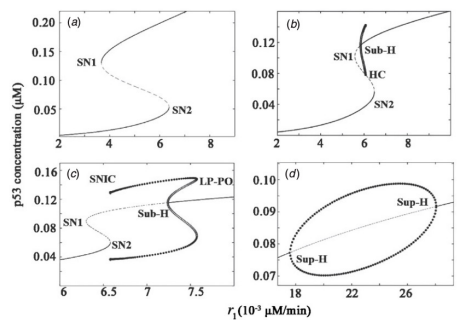
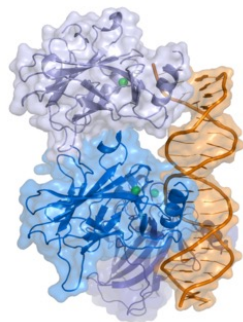
Physics



Phase transitions

Wikipedia; Labster.com

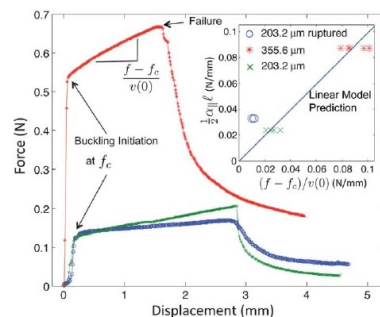
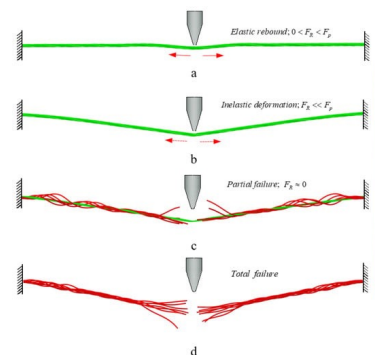
Medicine



Tumor metabolism

Wikipedia; Tingzhe Sun et al. *Phys Biol* (2010)

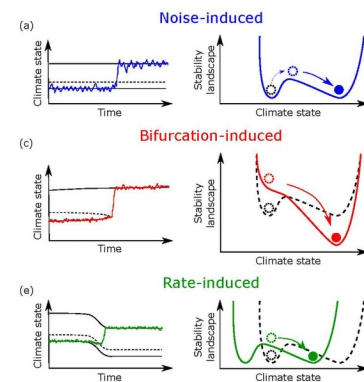
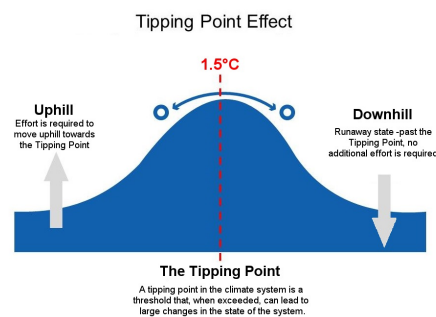
Engineering



Wire buckling events

Wu et al. *Buildings* (2023); Shan et al. *Soft Matter* (2013)

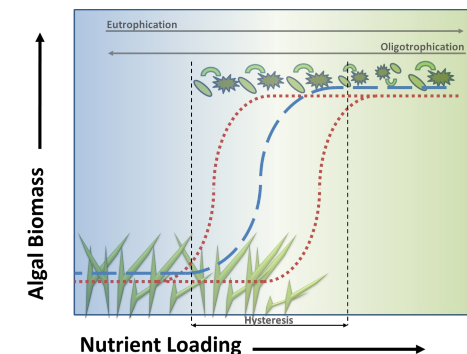
Climate science



Temperature tipping points

ClimateAtAGlance.com; Wikipedia

Ecology



Chlorophyll in lake ecosystems

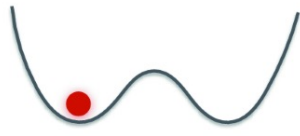
LakeScientist.com; Earth.com

How is the distance to criticality **typically measured**?

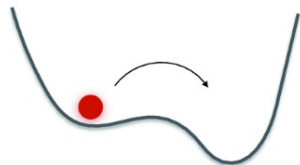
Critical slowing down

Dynamics are:
More variable
Evolving on a slower timescale

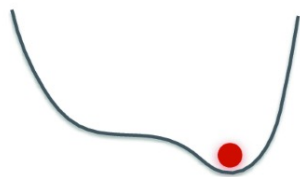
Far from transition



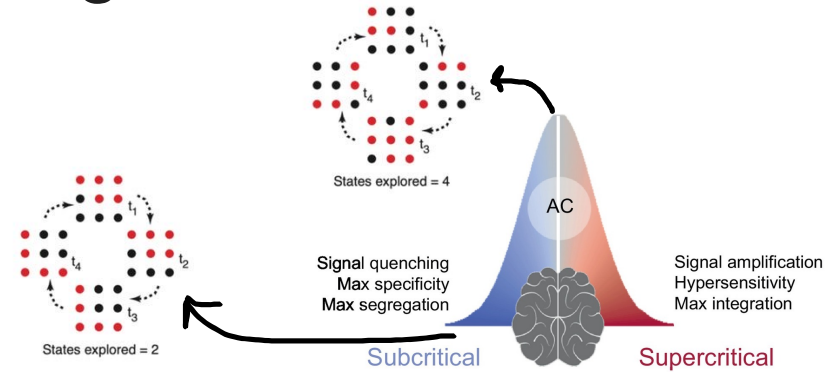
Close to transition



After transition



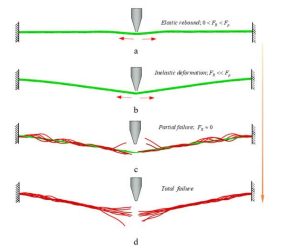
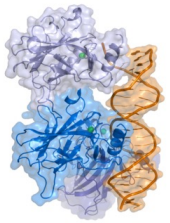
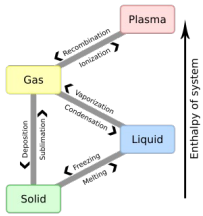
As you get closer to the critical point:



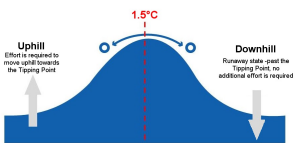
The system will explore a greater state domain closer to the critical point, which generally manifests as increase in the **standard deviation (SD) of the time series**.

Properties of the potential function naturally motivate time-series features like the **SD or lag-1 autocorrelation** to track distance to the critical point, but both these measurements are **biased by noise**.

Physical systems like the brain exhibit noise that is **not trivial** relative to the scale of their **deterministic dynamics**.



Tipping Point Effect



The Tipping Point

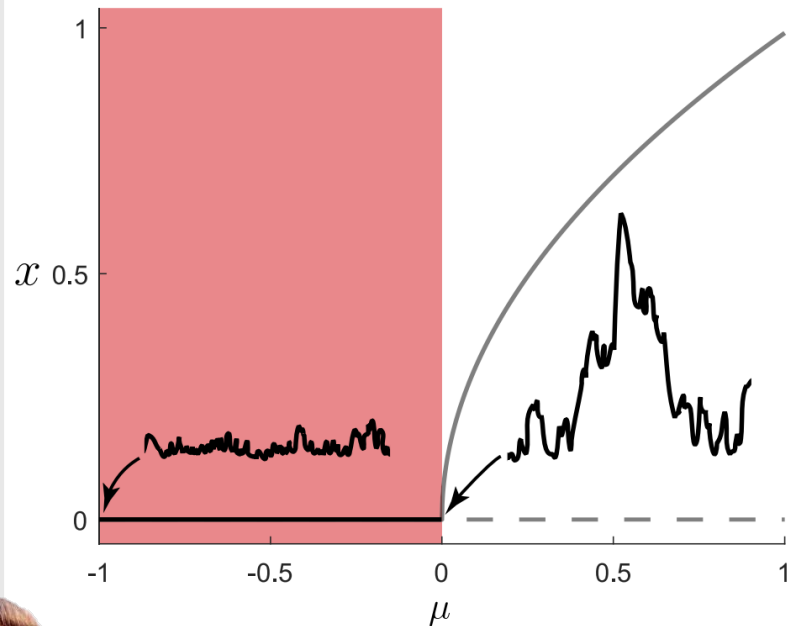
A tipping point in the climate system is a threshold that, when exceeded, can lead to large changes in the state of the system.



The search for a **DTC** measure that is **robust** across levels of **dynamical noise**

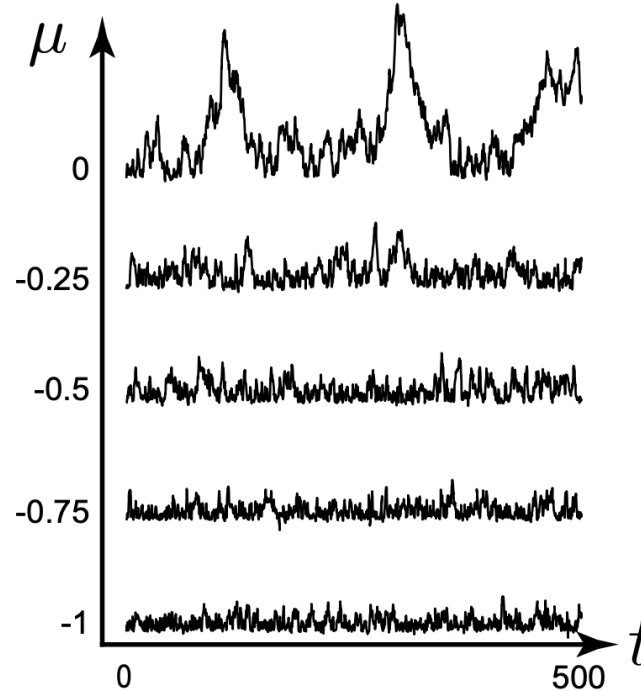
(a) Can we predict the DTC, μ , when uncertain noise corrupts conventional features?

$$dx = (\mu x - x^3) dt + \eta dW, \quad x \geq 0$$



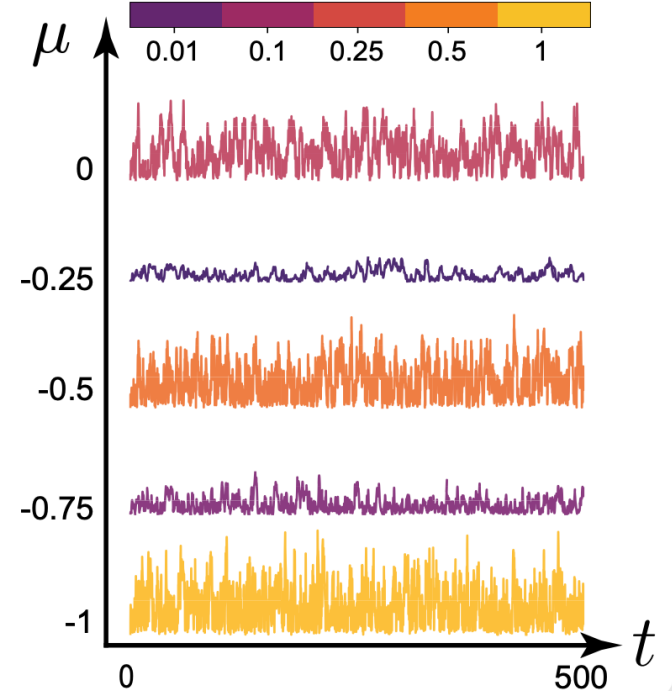
(b) Simulate time series for a range of μ

$$\eta = 0.05$$

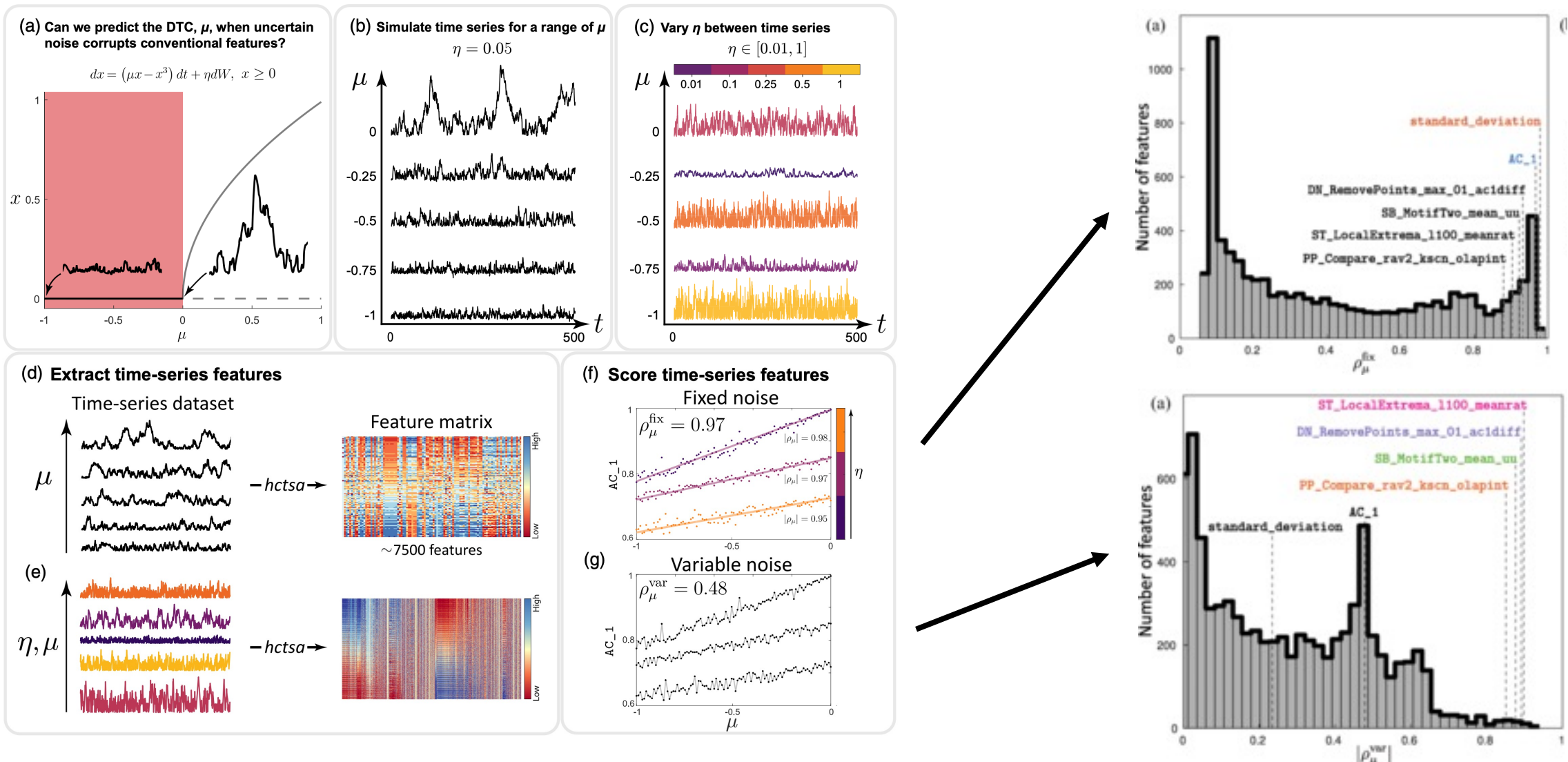


(c) Vary η between time series

$$\eta \in [0.01, 1]$$

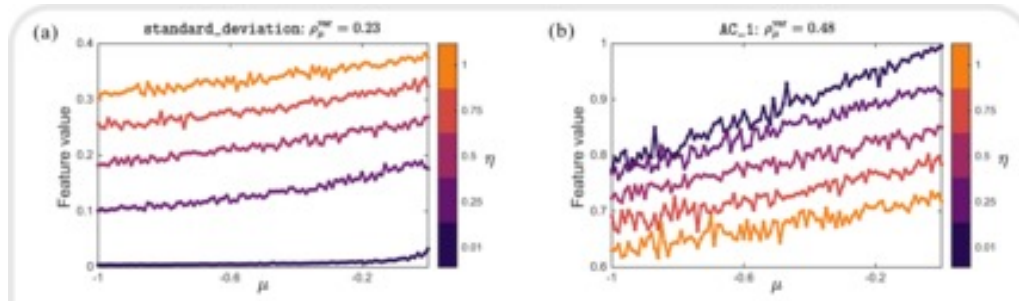


The search for a **DTC** measure that is robust across levels of dynamical noise

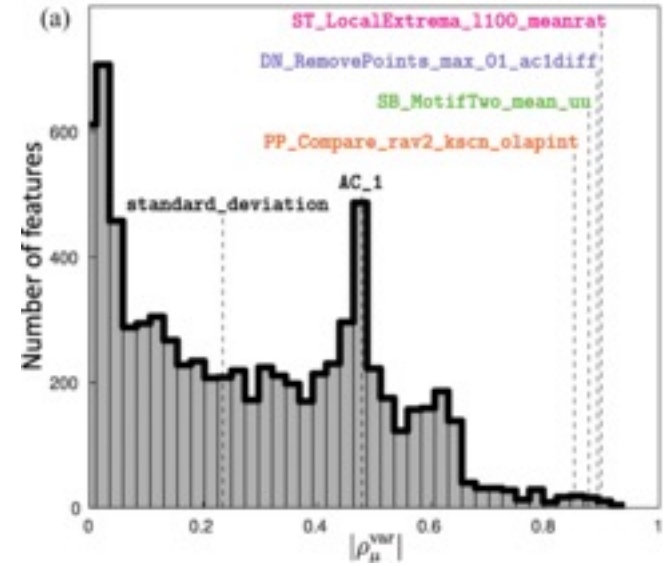


Conventional DTC measures are sensitive to noise

SD

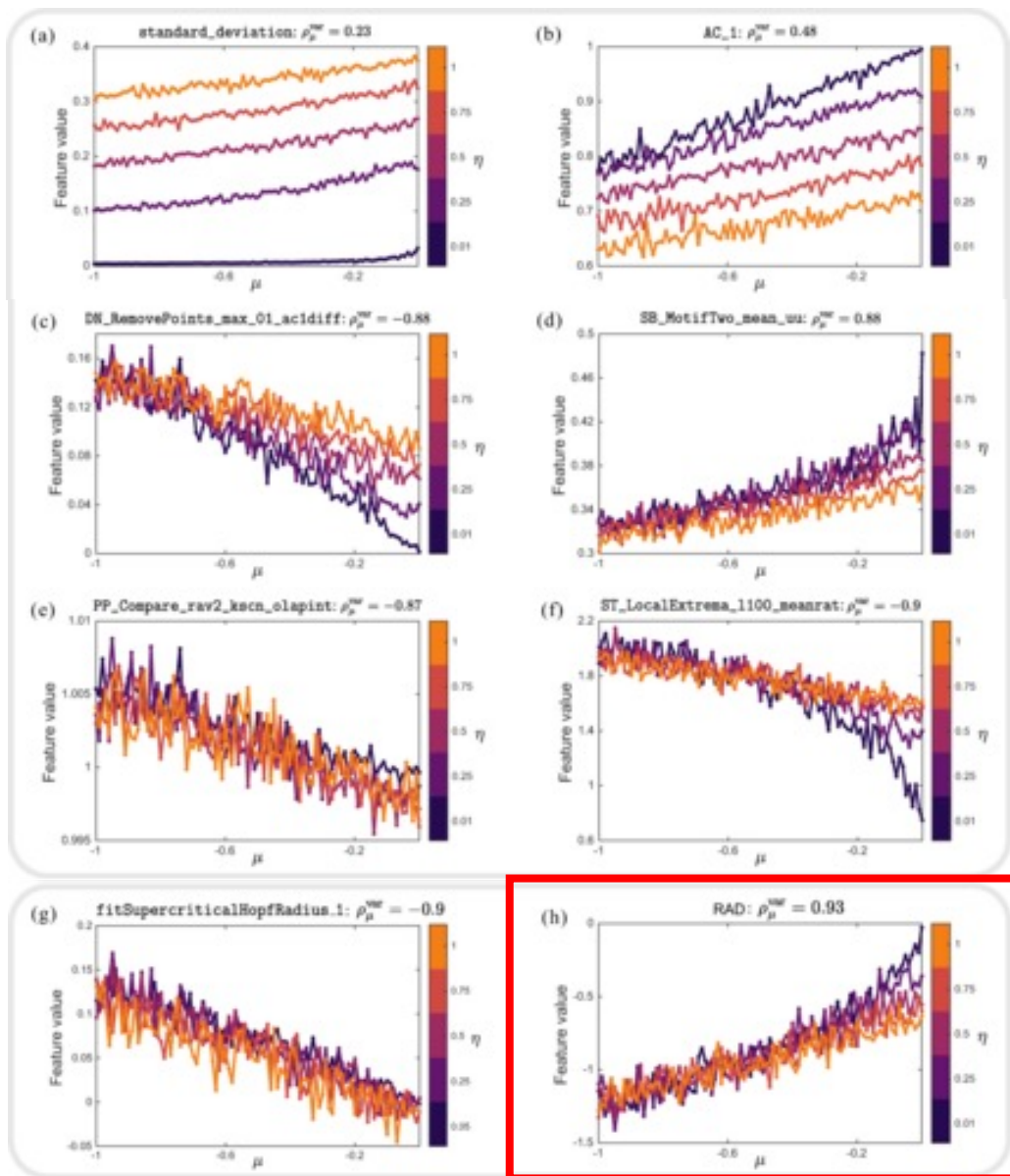


AC1

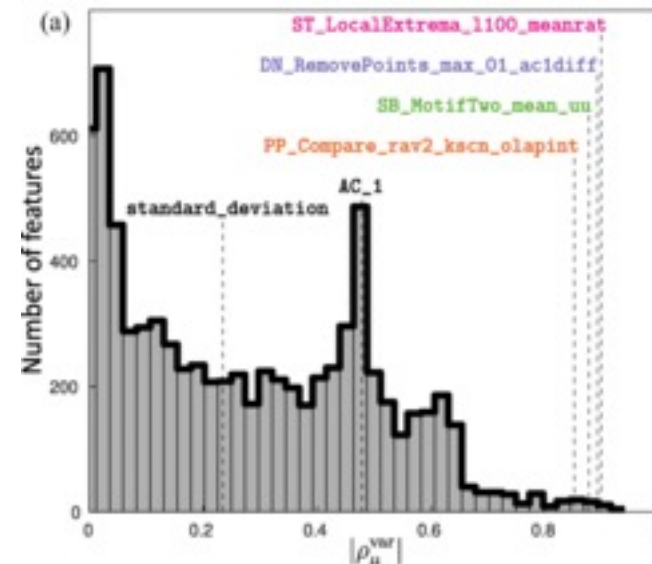


...but new noise-robust time-series features enter the villa

SD

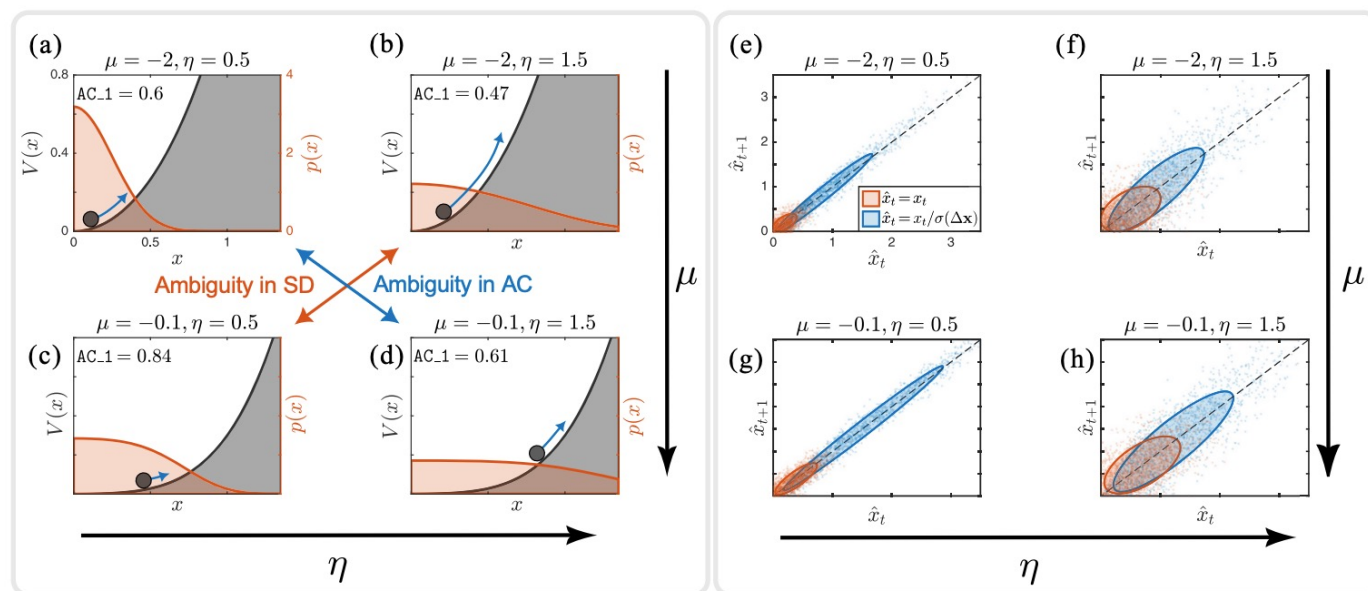
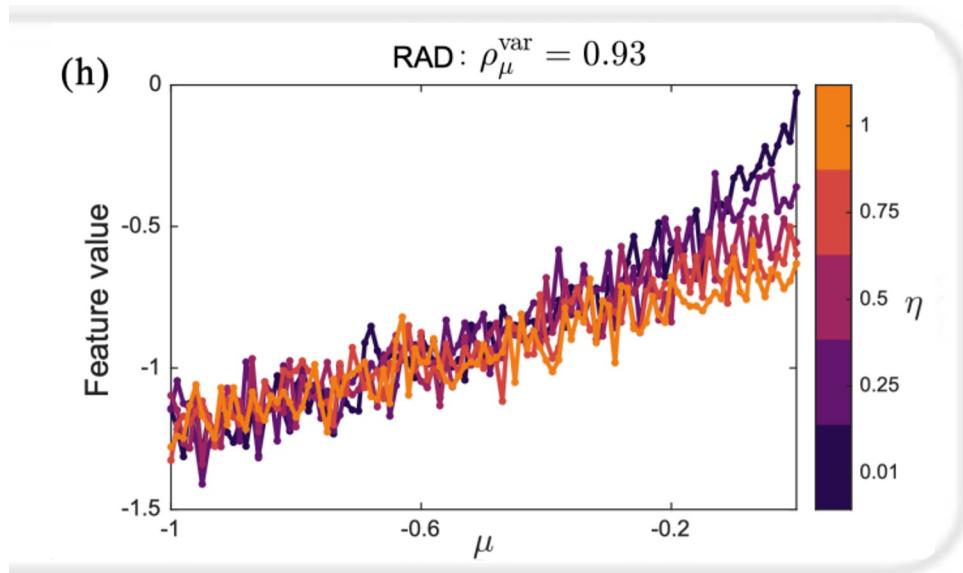


AC1



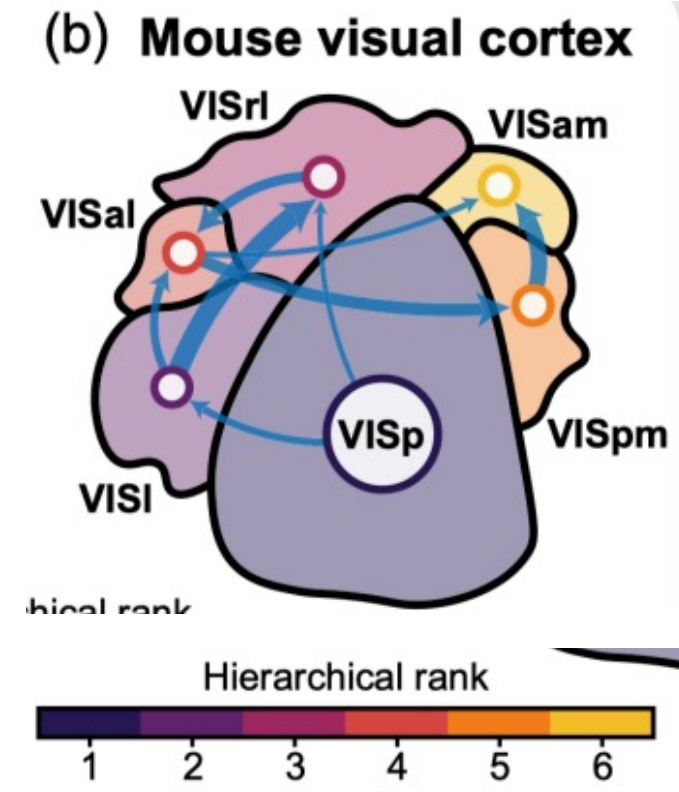
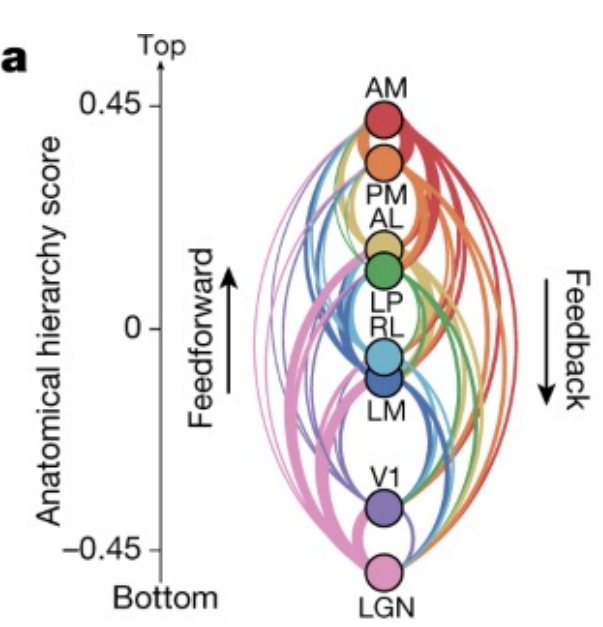
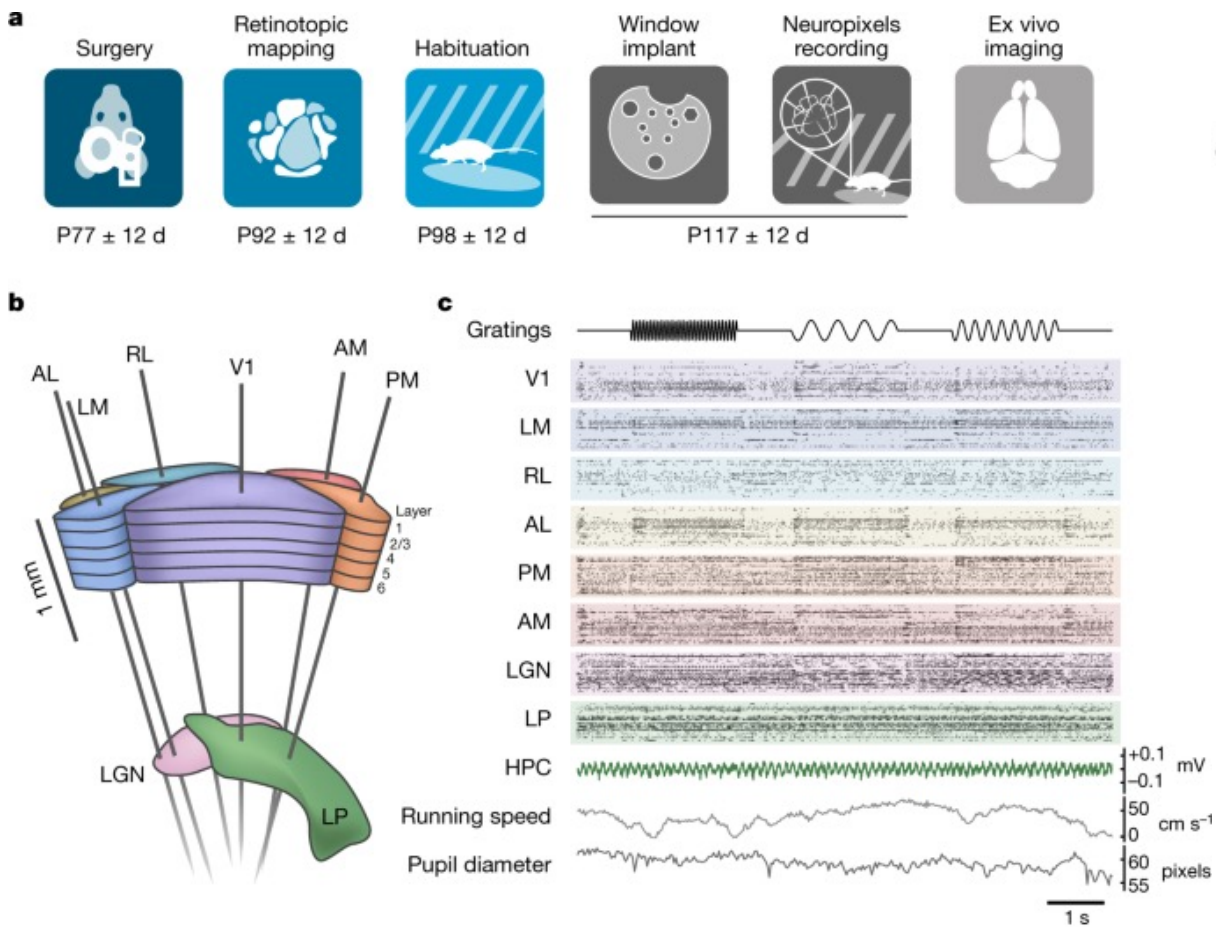
Brendan & co inspected properties of these noise-robust algorithms (identified via **data-driven analysis!**) to propose a new noise-robust index of the distance to criticality: **Rescaled autodensity**

...but new noise-robust time-series features enter the villa



Rescaling time-series values by the spread of differenced values corrects for the confounding effect of a **variable-noise amplitude**, by capturing the **shape of the invariant density** (which depends on both the DTC and the noise amplitude) relative to the **spread of fast fluctuations** (which depends only on the noise amplitude)

Application: Does **distance to criticality** vary across the **visual hierarchy**?

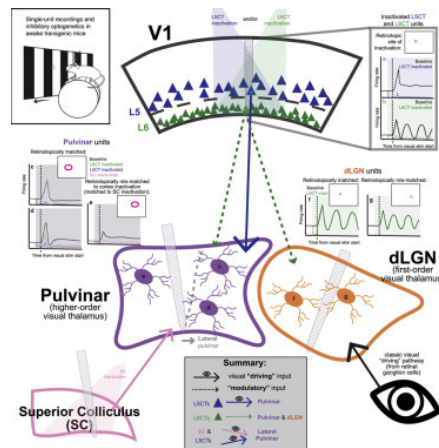
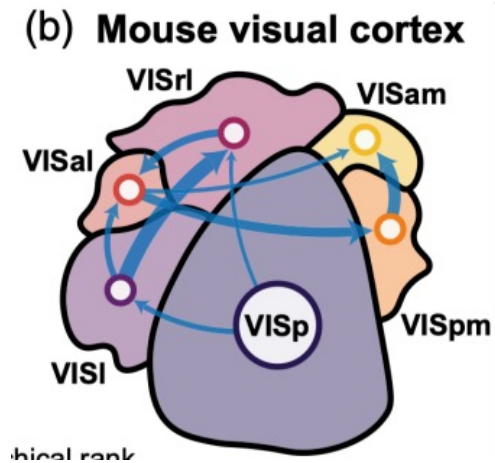


Application: Does **distance to criticality** vary across the **visual hierarchy**?

Hypothesis: **higher-order** regions exhibit **longer timescales** of neuronal activity because they are **closer to the critical point**

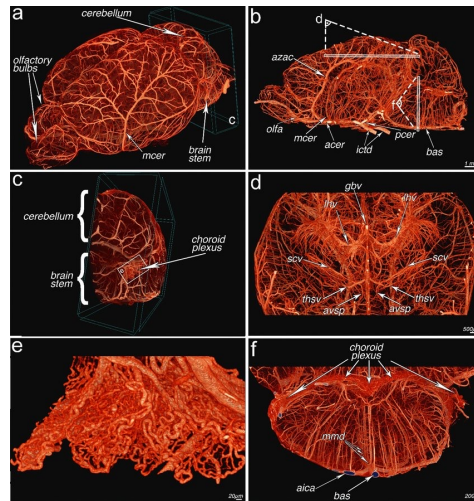
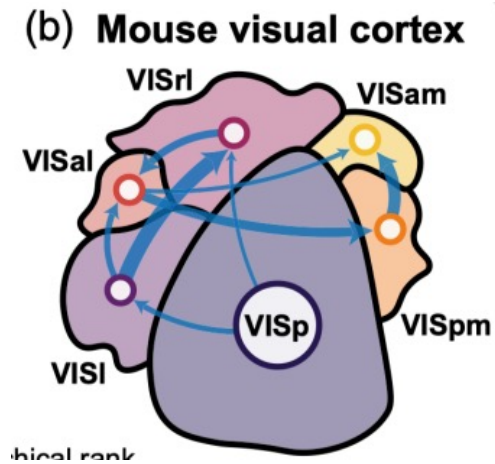
This is a good setting to test the noise robustness of RAD because we see variable noise levels across brain regions, attributable to:

- Differences in **thalamic drives**



Application: Does **distance to criticality** vary across the **visual hierarchy**?

Hypothesis: **higher-order** regions exhibit **longer timescales** of neuronal activity because they are **closer to the critical point**

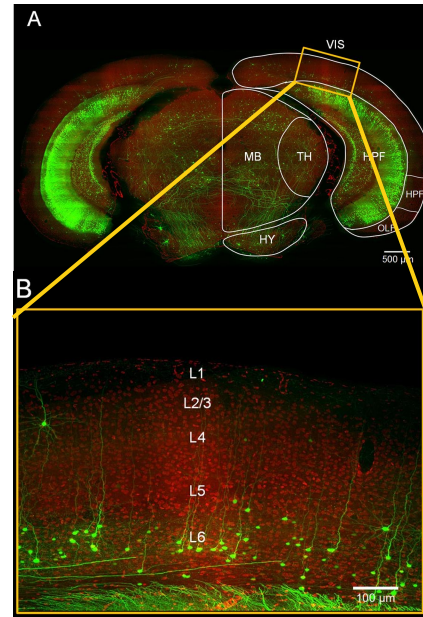
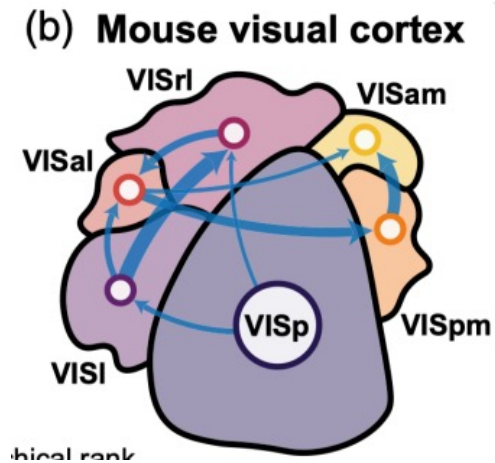


This is a good setting to test the noise robustness of RAD because we see variable noise levels across brain regions, attributable to:

- Differences in thalamic drives
- Differences in **adjacent vasculature**

Application: Does **distance to criticality** vary across the **visual hierarchy**?

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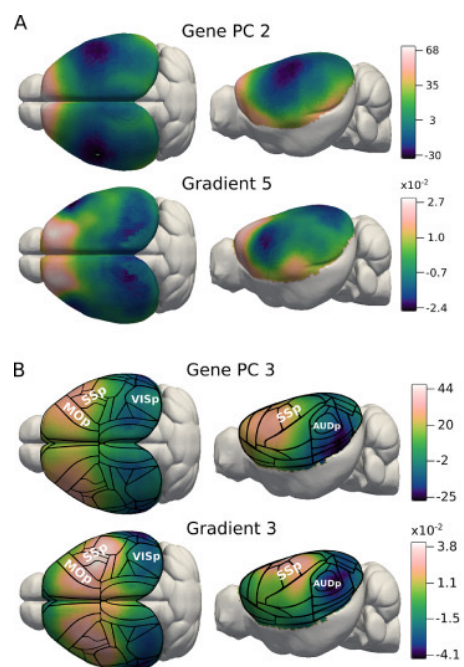
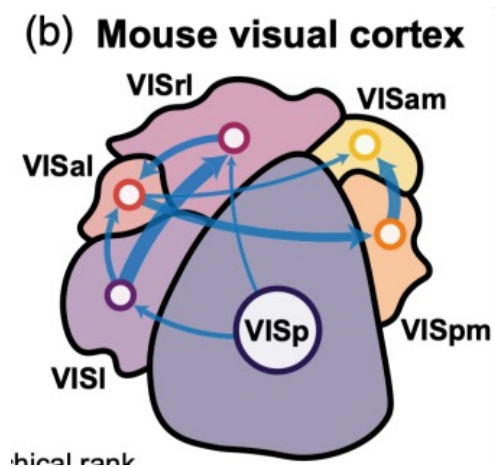


This is a good setting to test the noise robustness of RAD because we see variable noise levels across brain regions, attributable to:

- Differences in thalamic drives
- Differences in adjacent vasculature
- Differences in **cytoarchitecture**

Application: Does **distance to criticality** vary across the **visual hierarchy**?

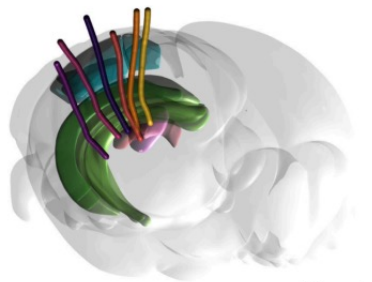
Hypothesis: **higher-order** regions exhibit **longer timescales** of neuronal activity because they are **closer to the critical point**



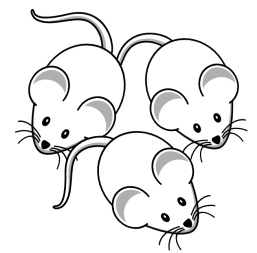
This is a good setting to test the noise robustness of RAD because we see variable noise levels across brain regions, attributable to:

- Differences in thalamic drives
- Differences in adjacent vasculature
- Differences in cytoarchitecture
- Differences in **transcriptomic and neuromodulatory gradients**

Application: Does **distance to criticality** vary across the **visual hierarchy**?

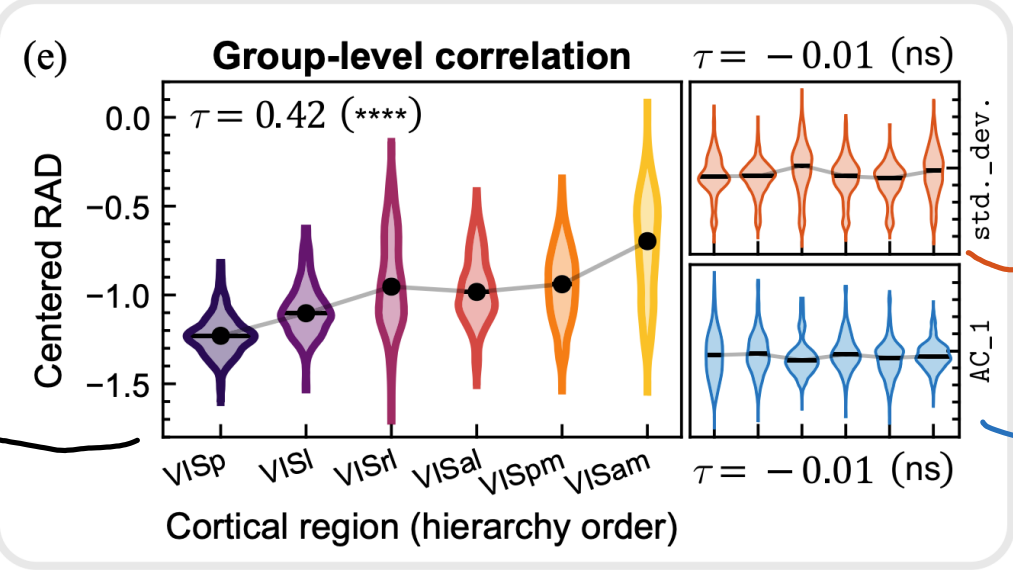
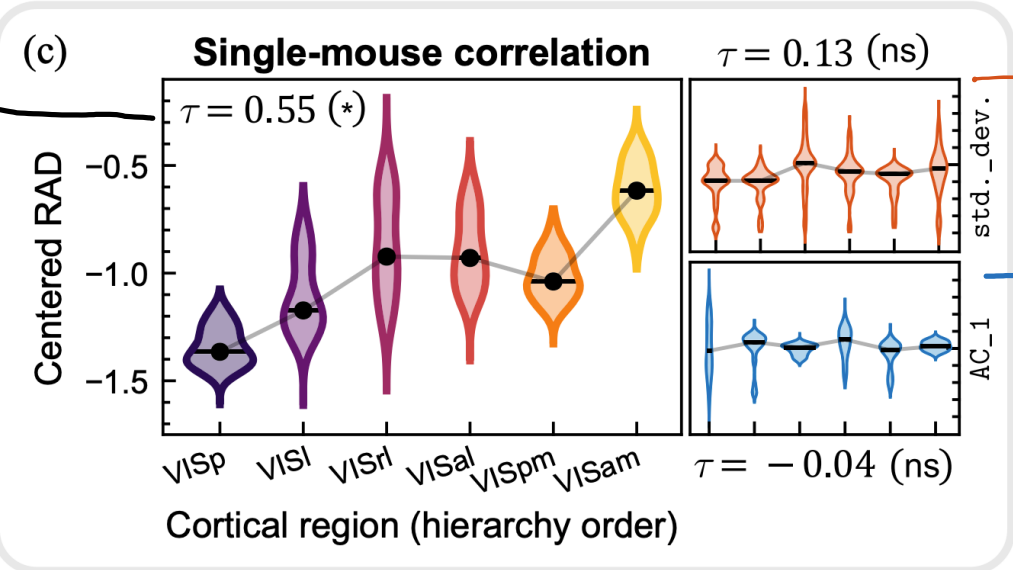


6 NeuroPixels probes



39 mice

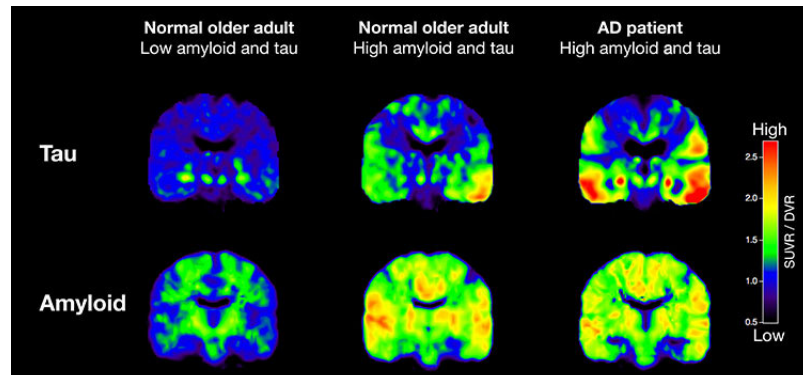
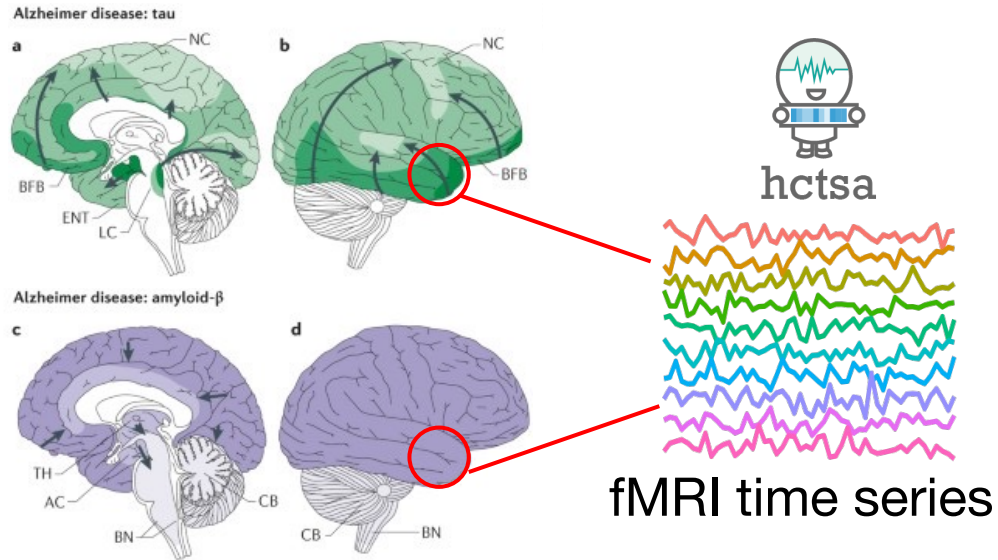
Across mice, RAD **successfully captures** the variation in **visual cortex hierarchy**



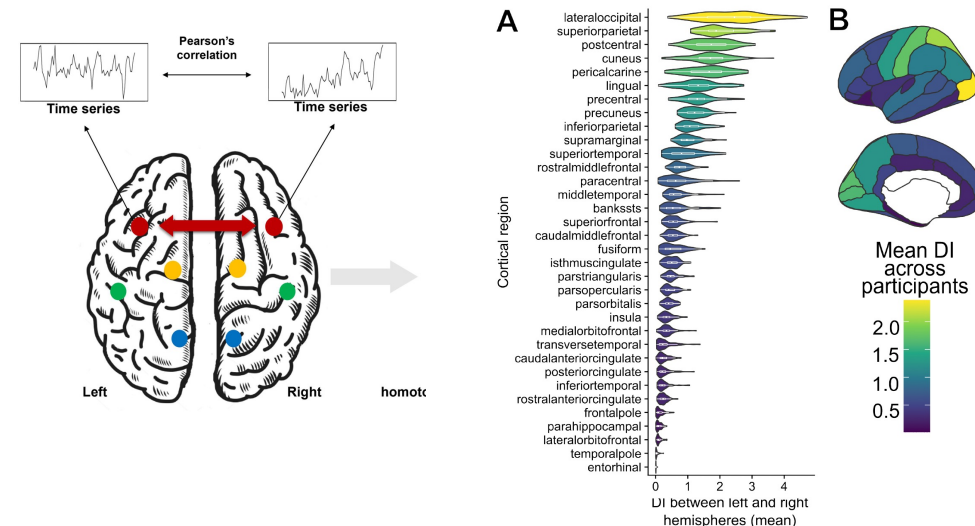
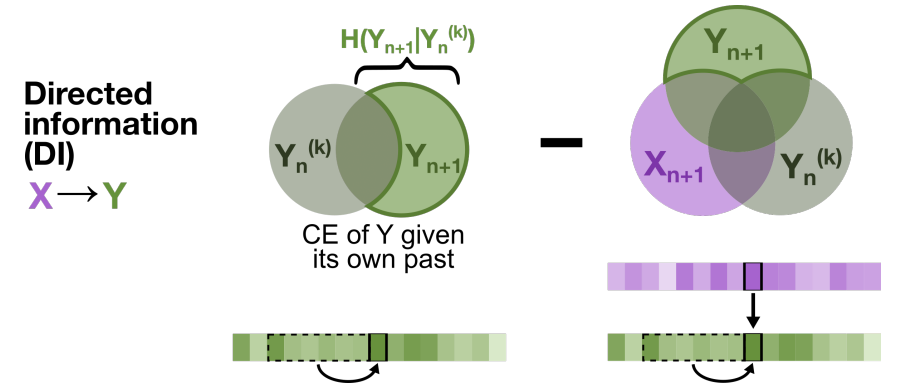
Traditional criticality metrics, SD and AC1, **fail to track** this hierarchical structure

My final PhD projects with hctsa + pyspi

Project #1: Characterizing the effects of Alzheimer's disease pathology on localized region-specific dynamics



Project #2: Directed information flow between the left and right hemispheres in health and disease



Thanks to my research groups as always 😊



Dynamics & Neural Systems Group

Dr Ben Fulcher

Trent Henderson

Kieran Owens

Aria Nguyen

Rishi Maran

Brendan Harris

Joshua Moore

Teresa Dalle Nogare



Shine Lab

A/Prof Mac Shine

Brandon Munn

Eli Mueller

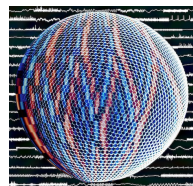
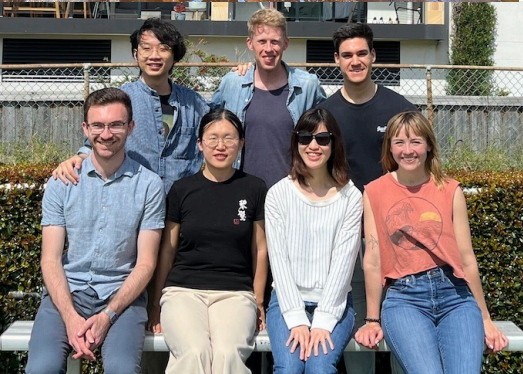
Natasha Taylor

Gabriel Wainstein

Christopher Whyte

Bella Orlando

Joshua Tan



THE UNIVERSITY OF
SYDNEY



[github.com/anniegbryant/
CSYS5040_Demo](https://github.com/anniegbryant/CSYS5040_Demo)

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